XVIII
Philolaus and the Formal Cause

(a) *Pythagorean numerology*

The Pythagoreans sailed their intellectual boats on the ocean of anonymity. One name stands out: Philolaus, according to a reliable tradition, was the first Pythagorean philosopher to publish his views; and his book *Concerning Nature* for the first time congealed the fluid oral tradition of the school (Demetrius, *apud* Diogenes Laertius, VIII.85=**44 A 1**).[1] A malicious and silly rumour insinuated that Plato in his *Timaeus* plagiarized the work of Philolaus (Timon, fr. 54 =**A 8;** Hermippus, apud Diogenes Laertius, VIII.84=**A 1**); if the gossip has a basis in truth, and Plato was influenced by Philolaus, then that adds an extrinsic interest to the book.[2]

Several fragments of Philolaus' book have been preserved. A majority of scholars has found them spurious, adding them to the vast library of pseudo-Pythagorean literature; but the arguments for scepticism are not very solid, and I am persuaded by those scholars who think that some at least of the texts are genuine productions of Philolaus' pen. It would be pointless to rehearse the published arguments, and I have no new thoughts to contribute to the debate: I shall proceed on the assumption of authenticity, and let the interested or sceptical reader prove the assumption himself.[3]

Philolaus is sometimes taken as a mere mouthpiece: the views he expounds are not his own inventions; they are the common wisdom of his fellow Pythagoreans. And it has been judged that Philolaus' book was 'unscientific and without real understanding of the doctrines it reports'; it reveals 'a thinker of no great stature, whose interest is peripheral'.[4] The later part of this chapter will, I hope, show that Philolaus is a philosopher of some merit; but before turning to that task I shall spend a few pages on Philolaus' anonymous colleagues whose views he allegedly parroted.

If Philolaus was an inaccurate parrot for Pythagorean views, then we need an accurate account of those views against which to measure his mouthings.[5] Such an account is to be found in Aristotle's *Metaphysics*. Aristotle's remarks on the Pythagoreans in *Met* A 5 are intricate and obscure; but three generalities can be essayed with some confidence. First, the Pythagorean views that Aristotle reports belong as a whole to the fifth century.[6] Second, Aristotle is not reporting a single philosophy, but several variations on the broad Pythagorean theme. Third, some of Aristotle's account bears a resemblance to the views expressed in the Philolaic texts.

Aristotle does not name Philolaus in *Met* A 5. Sceptical scholars think that the fragments are part of a post-Aristotelian production designed to repair and defend the Pythagorean philosophy which Aristotle has mauled; others think that the fragments are the wreckage of Aristotle's main source for Pythagorean doctrine.[7] I do not accept the former view, and I think that there are sufficient differences between the fragments and Aristotle's account to rule the latter out of court. For what it is worth, I imagine that Aristotle is reporting the major orthodoxies of Pythagorean thought, and that Philolaus

represents a heterodoxy: his heresy was, I suppose, deemed too slight by Aristotle to warrant special treatment. The matter lies beyond our knowledge; but it seems clear that we cannot justly interpret Philolaus' texts by way of Aristotle's reports. (And I shall spare the reader the profound *ennui* which an extended treatment of those reports would surely induce.)

The foundation and the distinguishing mark of the Pythagorean philosophy is number: according to Sextus

> The Pythagoreans say that reasoning [is the criterion of truth]—not reasoning in general but that which comes about from mathematics, as Philolaus said **(267: A 29)**.

Plutarch says that in Philolaus' view

> Geometry is the principle and mother-state *(metropolis)* of the other disciplines *(mathêmatôn)* **(268: A 7a)**.[8]

An old *acousma* runs: 'What is wisest?—Number'; and the primacy of number is a striking feature of Aristotle's account of Pythagoreanism (e.g. *Met* 985b23=**58 B 4;** 986a15=**58 B 5**). In the case of Philolaus himself, **B 4 (=280)** illustrates the same thesis; and Archytas, the leading Pythagorean of the generation after Philolaus, wrote this:

> The mathematicians seem to me to have attained a fine knowledge, and it is not absurd that they should think aright about each of the things that are; for, having a fine knowledge about the nature of everything, they were likely to have a fine discernment too about the particular things that there are **(269:47 B 1)**.

The question of Pythagorean mathematics is a notorious thing. Once upon a time, scholars gave the Pythagoreans most of the credit for the astonishing advances in mathematics made in Greece during the fifth century. Now a contrary scepticism is fashionable; and most, I guess, will assent to the judgment that 'in its essence, mathematics is not Pythagorean but Greek'.[9] Hippasus of Metapontum did not discover the irrationals; Pythagoras' theorem is not Pythagorean; and there was no great Pythagorean mathematician before Archytas of Tarentum.[10]

It is hard to dissent from that negative opinion; but it would be an error to infer from it that the Pythagoreans were not mathematically inclined. Aristotle's testimony is explicit:

> At the same time as these men [sc. the Atomists] and before them, those called the Pythagoreans touched on mathematics and were the first to bring them forward; and being brought up in them, they thought that their principles were the principles of everything (**270:** *Met* 985b23–5=**58 B 4**).

> The Pythagoreans, having devoted themselves to mathematics, and admiring the rigour of its arguments, because it alone of the studies men undertake contains proofs, and seeing it agreed that the facts of harmonics are due to numbers, thought that these and their principles were in general the causes of existent things **(271)**.[11]

Aristotle's testimony is backed by Eudemus, who ascribes a few of the theorems contained in our Euclid to the Pythagoreans (cf. **58 B 18, 20, 21**). In the case of Philolaus we have a general notice that he was well-versed in the mathematical sciences (Vitruvius, **44 A 6**), and detailed evidence of his work in harmonics (**B 6;** Boëthius, **A 26**). Philolaus' mathematical abilities were not, perhaps, great: a recent scholar accuses him of 'mathematical inconsistencies' and 'gross errors';[12] and we may well imagine that the Pythagoreans, as a group, were students rather than professors of the mathematical arts.

In any case, it is not for their technical but for their philosophical contribution to mathematics that the Pythagoreans win our interest. Aristotle **(270)** puts it very clearly: the Pythagoreans only 'touched on *(hapsamenoi)*' mathematics, in the technical sense; but they 'were the first to bring them forward *(proêgagon)*' in a philosophical context.[13]

What philosophical use did the Pythagoreans make of mathematics? The cynical will speak dismissively of number mysticism, arithmology, and other puerilities. And it is undeniable that a great quantity of Pythagorean 'number philosophy' is a 'number symbolism' of the most jejune and inane kind. According to Aristotle, the Pythagoreans 'say that things themselves are numbers' (*Met* 987b28=**58 B 13**), or that 'existent things are by imitation of numbers' (*Met* 987b11=**58 B 12**);[14] elsewhere he particularizes:

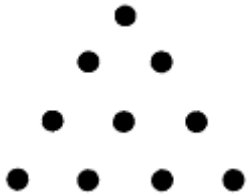
> The Pythagoreans, because they saw many of the attributes of numbers belonging to sensible things, assumed existing things to be numbers (**272:** *Met* 1090a20–22);

thus:

> Such and such an attribute of numbers is justice, such and such soul and mind, another opportunity, and so on for everything else (**273:** *Met* 985b29–31=**58 B 4**).

Alexander says that justice was 4, marriage 5, opportunity 7 (*in Met* 38.8–20); comparable assertions are attested for Philolaus;[15] and his younger contemporaries, Lysis and Opsimus, are said to have proclaimed that God is an irrational number (Athenagoras, **46 A4**).[16]

The Pythagoreans swore by the *tetraktus*. This was a graphic representation of the number 10:



And it exhibited in a vivid fashion some of the qualities of that number; for 'the number 10 seems to be perfect and to embrace the whole nature of number' (*Met* 986a8=**58 B 4**).[17] 'Touching on' arithmetic, the Pythagoreans were impressed by certain properties of the number 10; alas, their impression degenerated into a sort of mysticism: amazement, the nurse of philosophy, soon has her milk soured and turns into silly reverence and superstition. Those with a taste for intellectual folly will have their appetite sated if they go through the *TheologoumenaArithmeticae*. That Pythagorean work is a late compilation; the earliest examples of such symbolism are found in the *acousmata* and probably date from the time of Pythagoras himself: from first to last the Pythagoreans engaged in arithmology.

The mumbo-jumbo would not bear exposition but for the fact that certain Pythagoreans attempted to place a rational foundation beneath it.

> They believed that the elements of numbers are the elements of everything that exists (**274:** *Met* 986a1–2=**58 B 4**).

Aristotle's short statement can be illustrated from the *Pythagorean Memoirs* preserved by Alexander Polyhistor:

> The principle of all things is a monad, and from the monad comes an indefinite dyad, to play matter to the monad's cause; and from the monad and the indefinite dyad come the numbers; and from the numbers the points; and from these the lines, from which come the plane figures; and from the planes come the solid figures, and from these the perceptible bodies. (**275:** Diogenes Laertius, VIII.25=**58 B 1a**).

Alexander's account is influenced by Plato; but it is reasonable to believe that the Platonizing version is based on an earlier theory. Aristotle points to some such theory, and I assume that the fifth-century Pythagoreans did, in some sense, 'generate' the sensible world from the principles of number.[18] And that 'generation' would license or explain the crude assertions of arithmology: if horses, say, are ultimately 'generated' from the principles of numbers, then in an intelligible sense horses *are* numbers.

The 'generation' of things from the principles of numbers may, I fear, seem no less absurd than the primitive number symbolism I have just dismissed: how can men 'come from' numbers? How can abstract principles give birth to solid stuffs? If the 'generation' is construed literally, as a sort of cosmogony, then it surely is absurd; yet cosmogony is easily confused with analysis (witness Plato's *Timaeus*); and if we listen to the 'generation' system as a faltering attempt to play an analytical tune, unhappily transposed into the cosmogonical key, then we may hear something of modest interest.

The generation system becomes an abstract ontology. The thesis of this ontology is simple: the only ultimate entities in the world are the 'principles of number'. The ontology relies on three reductive analyses. First, the numbers can be reduced to a few basic principles. This rudely anticipates the insights of Leibniz and Peano: the number system can be built up from the unit (or monad) and the successor-operator (or 'indefinite dyad'). The ontology of arithmetic is reduced to a minimum. Second, geometry is arithmetized: the truths of geometry can be expressed in purely arithmetical terms; and geometrical objects can be constructed from numbers. That claim, I suppose, adumbrates the Cartesian discovery of analytic geometry. Finally, physical objects are reduced to geometry. There are two ways of effecting the reduction: first, each object has a characteristic shape; it is determined by, and can thus be identified with, some three-dimensional solid; second, the elemental stuffs which constitute the physical world are atomically structured, and their atoms have a characteristic stereometrical configuration. The former reduction will occupy us again; the latter is familiar from the *Timaeus*.

All truths of science are ultimately truths of arithmetic; all scientific entities are ultimately arithmetical. The generation system points to an ontological desert that is clean and arid even by the obsessively puritanical standards of American pragmatism; and at the same time it holds out the heady prospect of a rigorously mathematical approach to every branch of science. Yet if the Pythagorean ontology is stimulating, it is also wholly vague and programmatic; and I sympathize with the reader who remains unimpressed.

(b) *The philosophy of Philolaus*

Philolaus' book came to possess the traditional Ionian title *Concerning Nature*. And it seems probable that its contents followed the old Ionian models: we know that it elaborated an astronomy, a biology and an embryology (Menon, **44 A 28;** cf. **B 13**), and a psychology; and it is a plausible guess that it covered most of the traditional topics of the *phusiologoi*.

In a later chapter, I shall say something of Philolaus' psychology; here I may briefly describe his revolutionary astronomy. For Philolaus was the first thinker who dared displace the earth from its central position in the universe, and to suggest that, contrary to appearances, the earth was not stationary (Aëtius, **A 21**). In the Philolaic system, the centre of the cosmos was occupied by a mass of fire; around the fire circled the sun, the spherical earth, the moon, the planets, and that celebrated invention of Pythagorean astronomy, the *antichthôn* or counter-earth (cf. Aristotle, *Cael*, 293a17–27=**58 B 37;** Aëtius, **44 A 16**). The system contained a few grotesqueries. (The moon is inhabited, like the earth; and lunar creatures 'are fifteen times as powerful [as their terrestrial counterparts], and do not excrete' (Aëtius, **A 20**).[19]) Some judge it harshly: it was not 'a scientific astronomy' but 'a *mélange* of myth and *phusiologia*'; it was 'a superficial conglomeration of heterogeneous elements and naive speculation, not an attempt to find a deeper penetrating explanation of the phenomena'.[20] Those judgments are unfair: the fact is that we do not know what considerations led Philolaus to propound his startling innovations; and without such knowledge we cannot pass judgment. Astronomically, of

course, the Philolaic system is inadequate; but so are all the admirable astronomical systems of antiquity.

However that may be, Philolaus' views did not catch on. In the fourth century, Hicetas of Syracuse allowed the earth to move (Theophrastus, *apud* Cicero, **50 A 1**); but Hicetas' system was geocentric.[21] It was not till Aristarchus that the earth was again pushed from the centre of things; and since Aristarchus customarily wins credit for his heliocentric innovation, it is only decent to remember that the innovation was not an entirely unprecedented intellectual accomplishment.

If the superstructure of Philolaus' account of the world was Ionian in tenor, its foundations were characteristically Pythagorean.[22] The marriage of these two traditions (if I may change metaphors) was bound to produce curious offspring: how the consummation was effected must be discovered from the first six fragments of Philolaus' work.

Like the other neo-Ionians, Philolaus began by confronting the Eleatic challenge; and his starting point was, in one respect, even closer to Elea than theirs. For, like Parmenides, Philolaus approached metaphysics from epistemology: Parmenides' initial question was: What conditions must any object of scientific inquiry satisfy? Philolaus began by asking what things must be like if they are to be known; and the connexion between being and knowledge remains prominent in the development of his ideas.

According to Diogenes, Philolaus' treatise opened thus:

> Nature in the universe[23] was harmonized from both unlimited and limiting things—both the universe as a whole and everything in it **(276: B 1)**.

That initial statement was backed up by argument. It will be convenient to begin with **B 6,** which reads thus:

> And about nature and harmony things stand thus:—[i] The being *(estô)* of the objects, being eternal, and nature itself, admit divine and not human knowledge—[ii] except that none of the things that exist and are known by us could have come into being if there did not subsist *(huparchousas)* the being of the objects out of which the universe is compounded, both of the limiting things and of the unlimited, [iii] And since the principles subsisted being neither similar nor of the same tribe, it would have been thereby impossible for them to be arranged into a universe *(kosmêthênai)* if a harmony had not supervened, in whatever fashion it did come about, [iv] Now things that were similar and of the same tribe had no need of harmony; but those that were dissimilar and not of the same tribe and not of the same order (?)—it was necessary for such things to have been locked together by harmony if they were to be held together in a universe **(277)**.[24]

The text of **277** is in several places uncertain; and interpretation is always hard. I shall deal with sentences [iii] and [iv] later on; sentence [i] is a conventionally sceptical or

pious exordium (see above, p. 137): it is sentence [ii] which engages immediate attention.

The curious phrase 'the being of the objects *(ha estô tôn pragmatôn)*' must, I suppose, mean something like 'the existents *par excellence*'; at all events the phrase clearly denotes the same thing as 'the principles *(hai archai)*' of sentence [iii]. Of these principles we can know very little: first, that they are 'eternal'; second, that they consist of limiters and unlimiteds; third, that they require, in some cases at least, a harmonizing force. Why our knowledge is thus restricted Philolaus does not say: he implicitly rejects all Presocratic attempts to say what stuff or stuffs are primary, but he does so without argument. Perhaps he means only that a complete, and hence humanly impossible, knowledge of the present world would be required if we were to grasp just what types of principle were needed to generate it.

The eternity of the principles is presumably the *probandum* of sentence [ii]: 'the being of the objects subsists *(huparchein)*' means 'the principles are eternal'. The nerve of Philolaus' argument is constituted by two propositions:

(1) If *a* exists and is known to us, then *a* came into being.

(2) If *a* came into being, then the principles of *a* are eternal.

That (1) and (2) have an Eleatic background is plain enough; how precisely they relate to that background is a harder question to answer.

I take it that (1) is meant as an empirical observation: entities in the familiar world about us do, as a matter of fact, all have origins, near or remote. The epistemological *motif* which some have seen in (1) is only apparent: Philolaus does not mean that our knowing something requires that it be generated; he means only that the ordinary things that we do know are in fact generated.[25] Epistemology proper does not enter until **B 2** and **B 3**.

Premiss (2), on the other hand, fits easily into the box of neo-Ionian answers to Elea: things cannot come into being *simpliciter*, Philolaus avers, but they may spring from eternal, ungenerable and incorruptible, principles. The roots of Empedocles, the 'things' of Anaxagoras, and the atoms of Leucippus and Democritus are all eternal; and their eternity is generally regarded as a concession to Elea. Philolaus, in (2), makes an analogous concession; how useful these concessions are will be discussed in a later chapter.

Thus far nothing of a peculiarly Pythagorean character has emerged from **277:** it is the reference to 'limiters' and 'unlimited' things that gives the fragment its characteristic flavour; I shall approach this by way of **B 3** and **B 2:**

> For there will not even be anything that will be known if all things are unlimited **(278: B 3)**.
>
> [v] It is necessary for the things that exist to be all either limiting or unlimited or both limiting and unlimited, [vi] But they could not be only unlimited [or only limiting], [vii] Since, then, the things that exist are evidently neither from things all of which are limiting nor from things all of which are unlimited, it is clear then that both the universe and the things in it were harmonized from both limiting and unlimited things, [viii] And the facts too make this clear; for some of them, coming from

> limiting things, limit; and others, coming from both limiting and unlimited things, both limit and do not limit; and others, coming from unlimited things, are evidently unlimited **(279: B 2)**.

The logical form of Philolaus' argument is fairly clear. Let *P* abbreviate 'All existing things are limiting', *Q* 'All existing things are unlimited', *R* 'All existing things are both limiting and unlimited'.

Then [v] asserts:

(3) *P* or *Q* or *R*.

[vi] asserts:

(4) not-*P* and not-*Q*.

and [vii], inferring *R*, makes the further deduction that:

(5) Existing things were harmonized from both limiting and unlimited things.

Why does Philolaus expect us to assent to this curious argument? Premiss (3) is, I suppose, meant as an exhaustive disjunction, a logical truth. It is natural to read *R* as 'everything is both limiting and unlimited'; but that is ruled out by sentence [viii], which plainly places among 'the facts' the existence of some unlimited limiters and of some unlimiting unlimiteds. Hence if **279** is to be consistent, *R* must be read as: 'Some things are limiting and others are unlimited'. The reading is confirmed by the fact that it makes (3) a logical truth: the disjuncts are indeed logically exhaustive.

278, I take it, argues for not-*Q:* if we know anything, then not-*Q;* and we do have knowledge. The truth of **278** cannot be assessed until we have come closer to grips with the notion of a 'limit'. The first conjunct of (4), not-*P,* is a conjectural addition to the text; but not-*P* is plainly necessary to Philolaus' argument. No argument for not-*P* survives, but one is readily invented: surely if *a* is limiting, then *a* limits something; limiters logically require limitees. And again surely limitees are themselves intrinsically unlimited; if *a* limits *b,* then *b* is *per se* unlimited. But in that case the argument in 278 only needs a slight prolongment to prove not-*P* as well as not-*Q*.

The conclusion (5) is familiar from **276** and **277;** and it is the kernel of Philolaus' ontology. The thought that carries Philolaus from *R* to (5) is simple: what is itself a limiter cannot be compounded purely from unlimited things; no conjunction of unlimiteds will produce a limit. And conversely, what is unlimited requires unlimited constituents: a set of limiters will never give the unlimited its constitutional freedom. Thus (3) is a truth of logic; epistemology guarantees (4); (3) and (4) yield *R* by elementary logic; and *R* produces (5).[26]

(c) *Shape and number*

The 'facts'[27] alluded to in sentence [viii] are intended to convince us of the truth of (5): their form is logically appropriate but their content is obscure. Indeed, I fear that the fastidious reader will long ago have given up Philolaus in distaste: perhaps **279** contains a formally clear argument; but its substance is certainly misty and probably mystical. If that natural and entirely commendable feeling is to be dispelled we must discover what Philolaus has in mind when he talks of 'limiters' and 'unlimiteds'.

The fragments give no elucidation and no concrete illustration of 'limiters' and 'unlimiteds'; and the slim doxography is helplessly silent. Some scholars point to a notorious passage in Plato's *Philebus* which speaks of limits and the unlimited;[28] but that dialogue's gross obscurities give no help to a mind puzzling over Philolaus. Others read infinite divisibility into the 'unlimited' and speak of limiting atoms; but that will hardly fit the text. We are reduced to conjecture; but conjecture is not difficult, for an obvious interpretation is to hand: to apply a limiter to an unlimited is to give specific shape or form to a mass of unformed stuff. The 'facts' appealed to in [viii] will then consist of elementary examples of that type of operation: a potter moulds a wedge of clay into a pot; a sculptor casts a mass of bronze into a statue; a baker pats his dough into a loaf; a carpenter shapes a table from rough timber: all these artists apply a shape to a stuff, a limiter to an unlimited. Shapes are essentially limiting: anything shaped in such and such a way has, *eo ipso,* limits beyond which it does not extend; it is determined and circumscribed by its shapely boundaries. Stuffs, on the contrary, are essentially unlimited; clay and bronze, dough and wood, have no shapes. Any particular parcel of clay does, of course, possess some shape, however irregular or unaesthetic; but clay as such has no shape: 'What shape is clay?' is a nonsense question.

If we look at 'the facts', we find an abundance of cases in which things 'come from both limiting and unlimited things'; and they 'both limit and do not limit', i.e., they are compounds of a limiting shape and an unlimited stuff. But the 'facts' are also supposed to give us examples of compounds made exclusively from limiters, and of compounds made exclusively from unlimiteds. The former set of examples must, I imagine, be geometrical: a geometer may construct a square by conjoining two triangles, or a cube by adding two pyramids. Here two limiters are put together, and the result is a limiter; two shapes, conjoined, yield a third shape. Unlimiteds, too, are compounded: a metalworker may pour copper and tin together to make bronze; a cook mixes oil and vinegar; a painter blends one pigment with another. Such familiar operations are compoundings of one stuff from other stuffs, of one unlimited from other unlimiteds.

That interpretation seems to me to fit the Philolaic texts better than any other; and it gives Philolaus an original and important role in the development of philosophy.[29] The early Ionians, as Aristotle rightly insists, concentrated their attention on 'the material cause'; they inquired into the stuff of the universe, and supposed that one or two fairly simple operations on that *Ur*-stuff would suffice to generate our well-formed world. Empedocles and Anaxagoras also focussed their minds on matter: it was the diversity of stuffs rather than the diversity of substances which drew their attention and which they aspired to vindicate in the face of Eleatic objections. Atomism, it is true, pays some attention to form: the atoms have shapes, and are indeed referred to as *schêmata* or *ideai;* but there is no evidence that the Atomists placed any particular stress on the diversity of forms in the world, or that they went out of their way to account for the shape as well as the stuff of things.

Philolaus stands in strong contrast to that long tradition: he recognizes stuffs, but he insists equally on shapes. His fundamental tenet, expressed at the outset of his book in **276,** is that both matter and form are required in any analysis or explanation of the phenomena; we have to account not only for the diverse materials present in the mundane world, but also for the diverse ways in which those materials present themselves to us: we live in a material world, but the material is informed. And that,

after all, is the essence of Aristotle's judgment on the Pythagorean contribution to natural philosophy: they 'began to talk about what a thing is, and to make definitions' (*Met* 987a20=**58 B 8**); in other words, they began to investigate form as well as matter.

'But', Aristotle continues, 'they treated the issue too simply.' To see how Philolaus treated the issue we must look at two further fragments:

> And indeed all the things that are known have a number; for it is not possible for anything to be thought of or to be known without this **(280: B 4)**.[30]

> Number indeed has two proper kinds, odd and even [and a third from both mixed together, even-odd];[31] and of each kind there are many forms *(morphai)* which each thing in itself signifies **(28: B 5)**.

These two fragments stand in an intelligible relationship to **279**. The two 'kinds' of number are the odd and the even; and a strong tradition connects limit with odd numbers and unlimitedness with even numbers: in the *Metaphysics* Aristotle briefly delineates two Pythagorean views:

> These evidently believe that number is a principle…and that the elements of number are the even and the odd, and of these one is unlimited, the other limited; and the unit is from both these (for it is both even and odd)…. Others of the same group say that there are ten principles, set out in a column:
>
> limit and unlimited
>
> odd and even
>
> one and plurality
>
> right and left
>
> male and female
>
> resting and moving
>
> straight and bent
>
> light and darkness
>
> good and bad
>
> square and oblong
>
> (**282:**986a15–26=**58 B 5;** cf. Aristotle, fr. 203).

There is much in that column of 'principles' to excite the curiosity. Here I observe simply that odd associates with limit, even with lack of limit. (And there are explanations, of a vaguely arithmetical sort, for those associations.[32])

It is easy to suppose that Philolaus, who has limiting and unlimited principles, and who refers to the two 'kinds' of number, made the same association between the

members of these two pairs: Philolaic limiters are odd numbers; Philolaic unlimiteds are even numbers. I do not believe the interpretation. The main argument against it is that it does not, as far as I can see, lead to any clear overall understanding of Philolaus' theory of principles, whereas the alternative interpretation which I shall shortly offer gives Philolaus a fairly coherent philosophy. Two small points tell in the same direction: first, **280** suggests that 'having a number' is a sufficient condition for knowability; but if even numbers characterize the unlimiteds, then the unlimiteds too will be knowable—*contra* **278**. Second, the numbers, both odd and even, are said to be 'forms *(morphai)*'; that surely connects having a number with having a shape; but 'the unlimiteds' have no shape. I conclude that Philolaus differs from those Pythagoreans who assimilated odd and even to limited and unlimited. (That, indeed, is my chief reason for doubting that Philolaus was a main source for Aristotle's account of fifth-century Pythagoreanism.)

The 'forms' of **281** are presumably the natural numbers themselves: 2, 4, 6…are the forms of the kind *even;* 1, 3, 5…are the forms of the kind *odd*. 'Each thing in itself signifies' one of the natural numbers in that each thing is essentially determined by a natural number: what is known must have or be a limit or form; forms are expressed by numbers; hence whatever is known 'has a number'.

An explicit account of this sort of thing is ascribed to Eurytus, a pupil of Philolaus (cf. Iamblichus, **45 A 1**). Archytas told how Eurytus 'used to set out some pebbles, and say that *this* is the number of man, *this* of horse, *this* of something else' (Theophrastus, *Meta-physics* 6a19=**45 A 2**). Aristotle refers to the same practice (*Met* 1092b8=**45 A 3**), and a commentator explains it at length:

> Suppose for the sake of argument that the number 250 is the definition of man, and 360 of plant. Positing this, he used to take 250 pebbles—some green, some black, some red, and in general coloured in all sorts of hues; then, smearing the wall with plaster and sketching a man and a plant, he would stick these pebbles on the drawing of the face, these on that of the hands, others elsewhere, and he would complete the drawing of the pictured man by means of pebbles equal in number to the units which he said defined man (**283**: pseudo-Alexander, **45 A 3**).[33]

That sounds intolerably puerile; and puerile it doubtless was. Yet it is not quite as frivolous as it is sometimes imagined to be: Eurytus was not just 'drawing pictures with pebbles'; nor did his pebbles represent physical—or atomic—constituents of man.

Rather, he must have started from a geometrical observation: three points, however disposed, determine a triangle; and any triangle is determined by three points; four points determine a quadrilateral, and any quadrilateral is determined by four points. In general, then, geometrical and stereometrical figures will be determined by natural numbers; and since men and plants are stereometrical figures, they too will have their defining numbers. Eurytus' task was to work out 'the minimum number of points necessary to ensure that the surfaces formed by joining them would represent a man and nothing else';[34] and his pebble-dashing provided a striking if crude analogy to that grand scientific task. Philolaus, I assume, anticipated Eurytus; and in **278** and **279** we have the theoretical statement of the view which Eurytus' pebbles illustrate.

Is all this mere comical arithmology? or is it the first scrabbling essay towards a quantitative and mathematically-based science? Surely it is both of those things. Scientific theorems must be mathematical in their expression if they are to have the precision and utility we require of scientific knowledge: the early Milesian theories were largely unquantitative (above, p. 48), and their neo-Ionian successors seem to have done little better in that respect; even the Atomists made no attempt to apply arithmetic or geometry to scientific knowledge. Philolaus and Eurytus saw their failing, and attempted to meet it: the shapes of things are essential to them (we recognize things by virtue of their shapes); shapes can be expressed arithmetically; and the consequent arithmetical definitions of substances may be expected to function as the foundations of a mathematical physics.

In aim and scope the Philolaic project is admirable; in practice it is, inevitably, jejune. Shapes are not determined by natural numbers in the way Philolaus apparently imagined: does 4 determine a quadrilateral or a tetrahedron? does 8 determine an octagon or a hexahedron? Natural numbers alone will not do: if geometry is to be 'reduced' to arithmetic, the reduction must be carried out by more sophisticated means. Again, however important shapes may be in our recognition of substances, it is plain that they do not constitute the essence of substances. A poodle is not simply a mass of stuff formed in such and such a shape; it is a thing with certain powers and dispositions; decoy ducks and waxwork men, however cleverly modelled, are not ducks and men. Conversely, it is hard to imagine that there is *a* shape of man, let alone of dog or of plant: men come in different shapes and sizes; species of dog differ considerably in outline; and any attempt to distinguish the shape of a plant would be laughable.

Finally, stuffs have no shape—they are essentially unlimited; yet we surely do have knowledge of stuffs. Philolaus' fundamental assumption that 'there is no knowledge of the unlimited' seems to be a baseless prejudice; and it is implicitly contradicted by the third type of 'fact' to which the end of **277** appeals. No doubt genuine knowledge of stuffs must be in some sense quantitative: we do not have genuine knowledge if we only 'know' that tin and copper alloy to bronze; we need to know that a mixture of *n* per cent tin and *m* per cent copper yields bronze. But even if knowledge is thus connected with quantity and number, there is no connexion with shape or form, and we are left, it seems, with knowledge of 'the unlimited'. Some may feel that this point is at once so evident and so strong that it rules out my whole interpretation of Philolaus' philosophy. That feeling engages my sympathy; yet I still incline to accept the interpretation, and its consequent inconcinnity: no alternative fares any better, and Philolaus, I fear, is not wholly consistent or clear-minded.

(d) *The harmony of things*

Limiters and unlimiteds do not exhaust Philolaus' conceptual resources: these principles, by themselves, would not have sufficed for a universe 'if a harmony had not supervened, in whatever fashion it did come about' **(277)**; and the 'harmonizing' of the principles is adverted to again in **276, 279** and **B 7**.

'Harmony' translates—or rather transliterates—'*harmonia';* and the word, familiar to us from Heraclitus (see p. 600, n. 13), may mean no more than a conjoining or fitting

together. It is thus tempting to read no more than a tautology into sentence [iii] of 277: if there were no *harmonia*, then, quite trivially, limiters and unlimiteds could not have been fitted together. But sentence [iv] makes it plain that Philolaus meant more than that: *harmonia* is required not for any compounding, but for the compounding of things that are dissimilar or 'not of the same tribe'.

The dissimilar things are limiters and unlimiteds: why should conjunctions of limiter and unlimited require a *harmonia* when conjunctions of limiter and limiter, or of unlimited and unlimited, do not? Any two limiters may be fitted together: limiters are shapes, shapes are numbers, and any two numbers can be added together. What is more, their compound is eternally stable: the truths of arithmetic are indestructible. Again, most stuffs can be mixed or amalgamated into a moderately stable compound; such, at least, was the implicit assumption of all the Presocratic cosmogonies, and if a few trite examples tell against it (oil and vinegar proverbially separate), then either Philolaus ignored them or he supposed that they are not 'of the same tribe' even though they are 'similar': similar *qua* unlimited, they do not belong to the same kind of unlimiteds.

On the other hand, it is a clear empirical fact that not every shape can be fitted to every stuff: you may fashion a sphere of wood or metal, but you will not impose a spherical form on water or fire; the characteristic form of flames cannot, or cannot easily, be matched in wood; sand will form dunes but not pinnacles; mercury, globules but not cubes. Of the innumerable matchings of form and stuff that are possible, few are actual; and therefore some explanation is required for those matchings that do occur. In short, there must be a harmony between certain shapes and certain stuffs which accounts for their felicitous association. To say that there is a harmony is not to offer an explanation; it is to point out the need for an explanation. Just as the terms 'limiter' and 'unlimited' are schematic designations of types of principle, so the term 'harmony' is a schematic designation for a type of explanation: we cannot know what the essential nature of limiters and unlimiteds is; nor can we know how, in concrete terms, shape and matter cohere. What we do know is, first, that there must be both shape and matter; and second, that there must be an explanatory harmony of their conjunction.

Harmonia is not a static thing: it is introduced in a dynamic cosmogonical context. Nature 'was harmonized *(harmochthê)*' **(276);** the universe 'was compounded *(sunesta)*' **(277);** things were 'arranged into a universe *(kosmêthênai)*' **(277);** 'everything comes about *(gignesthai)* by necessity and harmony' (Diogenes Laertius, VIII.84=**A 1**). Two fragments of the cosmogony survive. One says merely that:

> The first thing to be harmonized in the middle of the sphere is called the hearth **(284: B 7)**.[35]

The other is longer; I quote it for its interesting attempt to deal with the notions of 'up' and 'down':

> The universe is one, and it began to come into being at the middle, and from the middle upwards in the same way as downwards. And what is upward is over against the middle from the point of view of those below; for to those below the lowest part is like the uppermost part, and

> similarly for the rest. For both have the same relationship to the middle except that their positions are reversed **(285: B 17)**.[36]

This fragment coheres well enough with the Philolaic astronomy, and it might easily come from a cosmogony in the traditional Ionian style; but the fact is that we know almost nothing of Philolaus' cosmogonical speculations.

A naive interpretation of Philolaus would imagine a pre-cosmic state of things in which on the one side there rose a vast mass of completely shapeless stuffs, and on the other side there stood a tailor's shop of forms: at the cosmogonie moment, something caused a suit to be taken from the shop and fitted harmoniously to the first fortunate lump of clay; and cosmogony proceeded, in orderly fashion, in the same general way, pre-existing forms being successively wrapped around suitable lumps of pre-existing stuff.

Aristotle makes two criticisms of the Pythagoreans which seem to tell against Philolaus even if they were not expressly aimed at him. First, he says that:

> They did not think that the limited and the unlimited and the one are different natures—e.g., fire or earth or something else of that sort, but that the unlimited itself and the one itself are the substance *(ousia)* of the things they are predicated of; and for that reason number is the substance of everything (**286:** *Met* 987a15–9 =**58 B 8**).

Second, he says that:

> These men evidently think that number is the principle for existing things both as matter and as affections and properties (**287:** *Met* 986a15–17=**58 B 5**).

Does not Philolaus in his cosmogony treat limiters, and hence numbers, as physical components of things? And does he not also treat the unlimiteds and the limiters as substances rather than attributes?

One part at least of the Aristotelian criticism does not touch Philolaus: he does not deny that the limiters and the unlimiteds are 'different natures'; that is to say, he does not assert that there are things which are *simply* unlimited and not unlimited fire or water or whatever. He does not imagine that the phrase 'the unlimited' picks out some peculiarly abstract kind of stuff; rather, he means that the original principles, whatever they are, are some of them limiters and some of them unlimited.[37]

The core of Aristotle's criticism, however, remains. Philolaus plainly holds, first, that limiters and unlimiteds are eternal, and second, that their cosmogonie harmonizing was an historical, or pre-historical, event. It follows that at some time there existed limiters or shapes that limited nothing or were shapes of nothing; and also that there existed shapeless, unlimited, masses of stuff. Moreover, the claim that the universe was 'compounded' from limiters and unlimiteds does powerfully suggest a picture in which the formal element in the compound is treated 'as matter'.

Some sort of a defence can be found for Philolaus: his shapes or limiters are, after all, essentially numbers; and the 'Platonist' view that numbers are eternal substances is

not to be abandoned merely on Aristotle's ukase. There are deep waters here on which Philolaus may, for a time at least, contrive to float. Again, Philolaus' pre-cosmic masses need not, perhaps, be literally devoid of form: Philolaus might have contented himself with the suggestion that the pre-cosmic form of stuffs was 'form' only in an etiolated sense—shape, but not intelligible shape, not mathematically determinable shape. Cosmogony, thus conceived, is the imposition of intelligible form on unintelligible matter. And finally, the crude conception of form as a quasi-constituent, and of information as a quasi-material colligation, can be purified or replaced by an unobjectionable notion.

I do not intend to follow up those vague suggestions: to do so would impose an anachronistic syncretism on Philolaus, uniting a Platonic account of mathematics with an Aristotelian position on form and matter. I prefer to end by underlining Philolaus' essential mistake. Rightly observing that a bronze sphere could be analysed into form (sphericity) and matter (bronze), Philolaus wrongly conflated that sort of analysis with the analysis of bronze into copper and tin. Bronze is compounded or put together from copper and tin; in much the same way, he supposed, a bronze sphere is compounded or put together from bronze and sphericity. In the latter case, to be sure, the components are 'dissimilar and not of the same tribe'; but the notion of compounding is the same in the two cases.

But the analyses and the compoundings are quite different: a chemical or physical analysis shows that bronze is made of tin and copper; a logical or conceptual analysis shows that a bronze sphere is made of sphericity and bronze. The former analysis, in Aristotelian jargon, breaks a thing down into its real parts, the latter into its logical parts: no physical process will separate the bronze from sphericity, and no logical penetration will reveal the chemical components of bronze. The distinction is not easy to articulate or expound; and the difficulties are increased by the fact that the same language is customarily used for both notions. Aristotle's commentators regularly fell into the confusion I am ascribing to Philolaus; and Aristotle himself only avoided it by the skin of his logical teeth.

Was Philolaus a great wit, or a ninny? We do not possess a vast amount of evidence, and the evidence we do have is of contested value. Certainly there are naive elements in Philolaus' thought; but equally certainly there are elements of bold originality, both in speculative science and in philosophy. I for one am prepared to credit Philolaus with the discovery of Aristotelian 'form'; and to claim that such a discovery was no insignificant achievement.