

XIII

Zeno: Paradox and Progression

(a) *Sprightly running*

Zeno's four arguments against motion are known to us from Aristotle's discussion of them in *Physics Z*. The Greek commentators on Aristotle for once fail us: they do not reproduce any of Zeno's own words, and with one trivial exception they provide us with no information we cannot glean from Aristotle's text. The paradoxes were famous in antiquity, and they influenced philosophers other than Aristotle: it is odd, as well as unfortunate, that our knowledge of them is virtually confined to the brief and polemical reports in the *Physics*.¹

Z 9 contains Aristotle's main discussion: the section begins thus:

There are four arguments of Zeno's about motion which provide difficulties for their solvers—first, the one about a thing's not moving because what is travelling must arrive at the half-way point before the end (we have discussed this earlier)...(**182: Phys 239b9–14=A 25=19 L**).

I shall refer to this first paradox as the Dichotomy; it is also called the Stadium, but I reserve that title for the fourth paradox.²

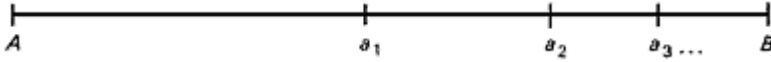
Aristotle's earlier discussion of the Dichotomy appears in Z 2:

That is why Zeno's argument assumes a falsehood—that one cannot pass through an unlimited number of things or touch an unlimited number of things individually in a limited time. For both length and time—and, in general, whatever is continuous—are called unlimited in two ways: either by division or as to their extremes. Now one cannot touch things unlimited in respect of quantity in a limited time, but one can so touch things unlimited by division—for the time itself is unlimited in this way. Hence it is in an unlimited and not in a limited time that, as it turns out, one traverses the unlimited, and one touches the unlimited things in unlimited and not in limited times (**183: Phys 233a21–31=A25=19L**).

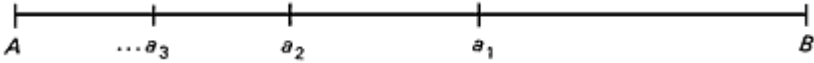
From this critical appraisal, and the half-line of description in Z 9, we must reconstruct the Dichotomy.

Suppose that in a finite period of time T , a body b traverses a finite distance AB . Then at some instant within T b will 'touch' a_1 , the mid-point of AB ; and at some later instant, again within T , b will 'touch' a_2 , the mid-point between a_1 and B ; and so on: in general, for any a_i there is a point a_j , mid-way between a_i and B , which b will 'touch' at some instant within T . Thus within T b will 'touch' infinitely many points along AB ; but that is impossible: hence b cannot traverse AB in T . And, in general, locomotion is impossible.

The exposition requires three preliminary comments. First, it interprets the Dichotomy in terms of a division which may be drawn thus:



Aristotle’s commentators prefer a different diagram:



Thus: ‘Before reaching B , b must touch the midpoint a_1 of AB ; and before reaching a_1 , b must touch the midpoint a_2 of $Aa_1\dots$ ’. Aristotle, I think, had the first diagram in mind;³ and I rest on his authority. But it is to be observed that there is no interesting logical difference between the two ways of expounding the paradox. Many commentators suppose that, with the first diagram, Zeno attempts to show that no moving body can ever *complete* its journey: it can never take the last step to B , since there *is* no last step. With the second diagram, they say, Zeno shows, more strikingly, that no body can ever *start* its journey: it can never take its first step, since there *is* no first step to take. But that distinction is unimportant: with either diagram, Zeno means to show that no body can traverse a finite distance in a finite time—in other words, that no body can move. The first diagram does not allow travellers to begin but not end their journeys; for every journey begun is *eo ipso* a journey ended.

Second, we may wonder what is meant by ‘touch’ (*haptesthai*) in Aristotle’s exposition, and why Zeno should think that moving bodies must ‘touch’ points in their travel. Some sources compare ‘touching’ to counting: b , they say, can no more ‘touch’ each a_i inside AB than he can count each a_i as he passes it; and some scholars have supposed that Zeno’s original traveller went on a mental rather than a physical journey: b cannot run through AB in his mind, for he cannot, so to speak, give his mind in turn to each of the points in AB . ‘Touching’, on this view, is a sort of mental stopping: the physical runner ‘touches’ a_i in so far as, on passing a_i , he stops mentally and says to himself: ‘This is a_i ’.⁴

That interpretation is surely not Zenonian: Aristotle explicitly distinguishes between Zeno’s paradox and the argument which requires the traveller to count the halfway points on his journey (*Phys* 263a6–11); and it is clear that the counting version was a vulgarization of Zeno’s argument. ‘Touching’ in Zeno is what it sounds like: physical contact. Let us idealize the example: suppose b to be a perfect cube travelling along the straight path AB ; and represent each a_i by a line across AB parallel to the front edge of b . Then b touches a_i if and only if a_i lies on the same plane as the front surface of b . Clearly, then, if b is to traverse AB , it must, in this sense, touch successively every a_i inside AB .

Third, we must be on our guard against the wiles of infinitude: in the paradox of ‘large and small’ Zeno falsely claimed an infinite regress; here does Zeno really show that b has infinitely many a_i s to touch? It is easy to show that he does: the successive a_i s are constructed by dichotomy: if $AB=1$, then $Aa_1=1/2$, $a_1a_2=1/4$, $a_2a_3=1/8$, and in general

$a_n a_{n+1} = \frac{1}{2^{n+1}}$. Thus the a_i s can be put in one-one correspondence with the powers of 2; hence they can be put in one-one correspondence with the natural numbers, and they are infinitely numerous.

Were he to travel from A to B , b would perform infinitely many tasks: there are infinitely many a_i s between A and B , each of which b must touch; there are infinitely many distinct propositions of the form ' b touches a_i ', and if b reaches B every one of these propositions has been made true. Thus Zeno thinks to establish:

(1) If anything moves, it performs infinitely many tasks. Since he holds it to be a truism that:

(2) Nothing can perform infinitely many tasks, he concludes that nothing moves. Unless we are to follow Zeno into his immobile world, we must reject either (1) or (2). Philosophical controversy has settled about (2); but I shall begin my discussion with a few thoughts on (1).

Some philosophers reject (1): in running from A to B , they say, b does not have to undertake, successively, infinitely many tasks; his run from A to B is not composed of an infinite sequence of progressively shorter runs. He performs one run, takes a hundred strides, feels fifty heart-beats, and so on; but he does not do or undergo an infinity of anything.

As an exercise in pure pedantry there is something to be said for that objection; for it is indeed mildly odd to call each of b 's successive moves to the next a_i a 'run' or a 'task'. Yet the point is merely verbal: Zeno himself does not use the terminology; and if b need not perform infinitely many tasks, that does not show that he has not got infinitely many a_i s to touch, infinitely many points to pass, infinitely many subsections of AB to traverse. The claim that b 's operations are not 'tasks', or are only 'tasks' in a Pickwickian sense, is boring.⁵

Proponents of the boring claim sometimes mean to deny (1) outright; but sometimes they offer a slightly subtler suggestion: b 's move from A to B , they suggest, is a *single* run, a *century* of paces, *half a century* of heart-beats, etc. To get from A to B requires the performance of many *finite* sets of tasks. Now, they continue, we can *describe* b 's journey in terms of an infinite succession of tasks; but that does not show (1) to be true: the performances required of a locomotor can indeed be described in terms of infinity, but for all that they are finite performances.

I set down this view because I have frequently heard it; but I dismiss it shortly: if any piece of locomotion can be truly described as the successive performance of an infinity of operations, then (1) is true. There are no two ways about it: either the description does not apply, or else it does apply and (1) is true; you cannot allow the description and brush aside the infinity it imports.

A more interesting objection to (1) can be formulated: (1) is true only if space is continuous or infinitely divisible; for it rests upon the assumption that any stretch of space, AB , contains infinitely many spatial points, a_i . That assumption, which I adverted to in the previous chapter (above, pp. 245–6), is not examined in our ancient texts; and those philosophers who do examine it usually accept it on insufficient grounds, tacitly supposing that a continuous geometry is applicable to physical space (and to time). Any full-scale examination of Zeno's paradoxes would be obliged to discuss the geometry of space: for reasons I gave earlier, I shall not enter upon such a discussion here.

(b) *Infinity again*

There have been numerous and diverse attempts to deal with premiss (2); the most convenient approach begins by asking wherein the impossibility of infinite performances is judged to lie: what is it about infinite performance in general, or the infinite performance of *b* in particular, that involves an impossibility? I shall mention seven lines of argument.

First: '*b* cannot run through infinitely many points; for that would mean that he traversed an infinite distance'. Many philosophers have faulted Zeno in the Dichotomy, as in the *logos* of 'large and small', for bad arithmetic.⁶ Let *AB* measure 1 mile; then in order to reach *B*, *b* must traverse $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$ miles. Now the sum of that series does not exceed 1 mile; but Zeno, it is alleged, supposed it to be infinitely great, and for that reason upheld (2).

If that was Zeno's reason for asserting (2), it was a tediously bad reason. And I am not confident that Zeno can be excused: if in the *logos* of 'large and small' he supposed the sum of $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ to be infinitely great, then he may well have made the same supposition in the Dichotomy. But whatever Zeno may have argued, others have found (2) plausible despite a degree of mathematical expertise; and we cannot dismiss the Dichotomy on the grounds that Zeno misreasoned for (2).

Aristotle's criticism of the Dichotomy in *Physics* Z 2 suggests a second way of defending (2): 'Locomotion must take a finite period of time: *b* gets from *A* to *B* in *T*; but in order to touch infinitely many *a*'s, *b* requires an infinite span of time.' Thus (2) is true because we are all hemmed in by the finitude of our lives.

Aristotle's reply is apt: just as *AB*, though finite in extent, contains infinitely many points, *a*_{*i*}; so *T*, though finite in duration, contains infinitely many instants, *t*_{*i*}. Each *a*_{*i*} in *AB* can be uniquely paired with a *t*_{*i*} in *T*. Similarly, every spatial sub-interval *a*_{*i*}*a*_{*j*} of *AB* can be uniquely paired with a temporal sub-interval *t*_{*i*}*t*_{*j*} of *T*. If space is infinitely divisible, so too is time; and thus in his run from *A* to *B*, *b* will never be short of time; there are as many instants available for touchings as there are points to be touched.

Aristotle assumes that time, like space, is continuous. And Zeno might be defended by denying that assumption: space is continuous, but time is not; unlike space, time consists of a succession of discrete *minima*. Time is granular, space is smooth: the parallelism between the two dimensions is broken, and with it Aristotle's objection. One of Aristotle's successors, Strato of Lampsacus, apparently held this view; and it has its modern supporters. I content myself with asserting that it is based on a bad lot of arguments, and that its acceptance involves a host of difficulties.⁷

In any event, that strange view is not the only way of defending Zeno: the second defence of (2) fails; but (2) does not fall with it: (2) has been upheld by men who are convinced of the continuous nature of time. Aristotle himself came to see that:

But this solution [sc. that of Z 2] is adequate with regard to the questioner (for he asked whether one could traverse or count unlimited things in a limited time), but it is not adequate with regard to the facts and the truth; for if someone were to forget about the length and about asking whether one can traverse unlimited things in a limited time, and

were to make this enquiry of the time itself (for the time has unlimited divisions), this solution is no longer adequate (**184: *Phys* 263a15–22**).

The observation that time as well as space is infinitely divisible only raises the further question of how we can endure through a finite stretch of time, if every such stretch contains limitless parts. Instead of one infinity to traverse, we have two: if *AB* is impenetrable, *T* is unendurable.

Aristotle is right: no Zenonian will be greatly moved by the solution of *Physics* Z 2. Aristotle himself suggests that we ignore *AB* and consider progress through *T* by itself; Zeno, I suspect, would have preferred to consider *AB* alone, without reference to *T*. Indeed, I am inclined to think that the reference to a finite time *T* did not occur in Zeno's original paradox: Zeno considered it impossible to touch infinitely many points; the impossibility is contained in the infinity of points, and factors of time are impertinent. Time is not, and need not be, mentioned in (2).⁸

After recanting his remarks in *Physics* Z 2, Aristotle essays a second solution to the Dichotomy; and his second solution suggests a third reason for upholding (2). Thus: 'If *b* is to touch infinitely many *a*_{*s*}, then an infinity of points must actually exist on *AB*; but there cannot *actually* be infinitely many existent points in a finite space.' That argument for (2) is of little intrinsic interest; but Aristotle's second solution to the Dichotomy warrants a moment's attention.

Here is the text:

Hence we must say when asked if it is possible to traverse unlimited things—either in time or in distance—that in a way it is, and in a way it isn't: if they exist actually, it is not possible; if potentially, it is. For someone moving continuously traverses unlimited things incidentally, not absolutely; for it is incidental to the line to be unlimitedly many halves, but its essence and its being are different (**185: *Phys* 263b3–9**).

That is an obscure paragraph. I take Aristotle to mean that *b* may touch infinitely many *a*_{*s*} provided that the *a*_{*s*} do not all actually exist. Thus (2) would be true if touching *a*_{*s*} involved the actual existence of *a*_{*s*}; but in fact, the *a*_{*s*} need only exist potentially, and that they may do.

All that, I think, has little effect on any honest Zenonian: Aristotle simply asserts that *b* can touch the infinitely many *a*_{*s*}, provided that he touches them 'incidentally' and they exist 'potentially'. The Aristotelian jargon only partially disguises the fact that Aristotle is offering a denial of, and not an argument against, premiss (2).

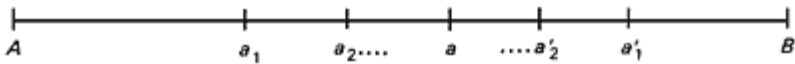
But there is something of interest here: a sufficient condition for the 'actual' existence of *a*_{*s*} is *b*'s stopping at *a*_{*s*}. According to Aristotle, *b* can touch the *a*_{*s*} as long as he does not stop at them, and hence actualize them. The keyword here is 'continuously (*sunechôds*): *b* is safe, Aristotle opines, if he runs smoothly, leaving the infinite *a*_{*s*} in their state of innocent potentiality. Tolstoy agreed: referring to the Achilles, which raises the same puzzle as the Dichotomy, he wrote: 'The ancients regarded, this as an unanswerable dilemma; its absurdity lies in the fact that the progress of Achilles is calculated on units with stoppage between, while it is in fact continuous' (*War and Peace*, bk 12, ch. XXII).

Consider a runner c who, like b , is set to traverse AB . c is allowed $2T$ for his performance; and he determines to produce a staccato run. Thus in $\frac{1}{2}T$ he runs from A to a_1 ; then he rests for $\frac{1}{2}T$ at a_1 . In the next $\frac{1}{4}T$ he runs to a_2 , and then rests for $\frac{1}{4}T$ at a_2 . In general, he takes, like b , $T/2^n$ to traverse the interval $a_{n-1} a_n$; but, unlike b , he rests for $T/2^n$ at each a_n .

Aristotle will allow b to reach B , but will deny c his goal; for c 's rest periods involve the impossible actualization of the infinitely numerous a_n s. And, of course, though we meet b every day, we do not come across staccato runners like c . Much modern discussion of the Dichotomy has in fact focussed on staccato performances; and a central problem has been to determine the truth of (2) if the performances are discrete or staccato in the way in which c 's is. We might, I think, intelligibly uphold (2) for c while rejecting it for b . Yet if that view seems reasonable, we have as yet no reason for accepting it. Aristotle's talk about actualized points is unconvincing; and no other argument yet allows us to distinguish between b and c in respect of (2). 'Intuition'—the kind name for untutored prejudice—favours b 's chances above c 's; but intuition is a fool's guide in this as in every other branch of philosophy.

Aristotle's double discussion of the Dichotomy is meritorious but not conclusive; and I turn now to the fourth of the seven reasons for maintaining premiss (2). It is comfortably simple: 'Infinite sequences of tasks have, by definition, no last member; and it is, trivially, impossible to complete a series of operations none of which is the last operation.'

Both premisses in this argument are false. The sequence actually employed in the Dichotomy does indeed lack a last member; but Zeno could easily have provided a sequence with both a last and a first member, as the following diagram shows:



If b is to traverse AB , he must touch each of the infinitely many points $a_1, a_2, \dots, a, \dots, a', a'_2$. His first step is from A to a_1 ; his last is from a'_1 to B .

As for the second premiss, it owes the little plausibility it has to an equivocation: you might, I suppose, say that the 'completion' of a series of tasks is simply the performance of the last member of the series; I complete the crossword in entering the last light, I complete the book in writing the ultimate sentence. And in *that* sense of 'complete' (if it is really a sense at all), sequences with no last member cannot be completed. But the obvious sense of 'complete' does not yield that consequence: if S is the set of tasks $\{x_1, x_1, \dots, x_n, \dots\}$, then b has completed S if and only if b has performed every x_i . To complete a set of tasks is to perform all the tasks, not to perform a *last* task. From ' b has performed every x_i ' we cannot infer ' b performed one x_i after all the other x_i s': even if S is finite, it need contain no last member; b may perform two, or three, or all of the x_i s at the same time.

That simple reflexion may still some disquiet about the propriety of speaking of 'completing' an infinite sequence of operations; but it will leave some readers unsatisfied. Surely, they will feel, even if you do not need to perform a *last* task in S in order to complete your performance, yet you cannot *complete* S without coming to an end of your tasks; and your tasks, being infinite, have no end. Thus (2) is true; and its

truth follows from the nature of completion: infinite tasks cannot, logically, be completed.

But that, too, is a bad argument; and it, too, trades on an equivocation. Someone who said ‘*b* cannot come to an end of his tasks’ might mean:

(3) $(\forall t)(\exists n)$ (if at *t* *b* has performed exactly *n* x_i s, then there is an x_j that *b* has not performed by *t*).

He might, alternatively, mean:

(4) $(\forall t)(\exists j)$ (has not by *t* performed x_j).

Now Zeno’s premiss (2) is equivalent to (4); and if *S* is infinite it is easy to demonstrate the truth of (3). But (3) and (4) are not equivalent; nor does (3) entail (4). By adding a further premiss:

(5) $(\forall t)(\exists n)$ (by *t* *b* has performed exactly *n* x_i s) it is possible to deduce (4) from (3); but to assert (5) is precisely to deny that *b* can perform an infinite number of tasks—and that is the very proposition at issue. In short, to say that you can never get to the end of an infinite sequence either crudely reasserts that you cannot perform infinitely many tasks, or else observes (truly but irrelevantly) that whatever finite number of tasks you have performed, more yet remain; neither assertion has any tendency to prove the truth of (2).

The next reason for supporting (2) turns on the notion of progress: ‘If I perform a sequence of tasks, my performance is marked by a certain type of progress. I gradually tick off the tasks to be done, reducing their number until eventually all are finished and behind me. But if *S* is infinite, I can make no progress of that sort; for however many x_i s I have performed, I still have exactly as many x_i s left to perform: however many a_i s *b* has touched, he still has exactly as many a_i s left to touch. Performance depends on progress; infinitude mocks progress: (2), then, is true.’

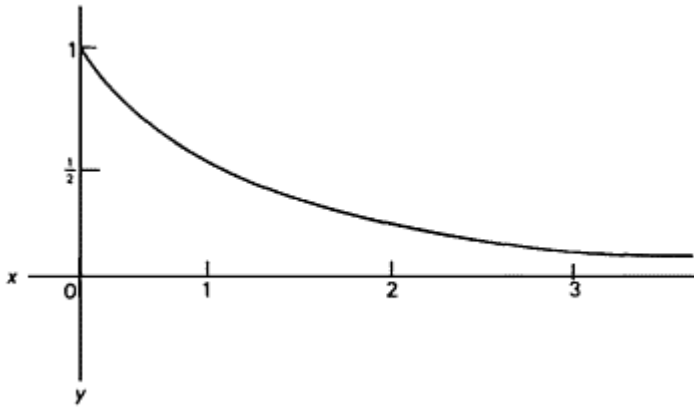
The second premiss of this argument is true: if *b* has touched *n* a_i s, he still has infinitely many a_i s to go. In general, if *S* has infinitely many members, and *S*’ is a finite subset of *S*, then *S*-*S*’ has as many members as *S*. (The series $\langle n, n + 1, n + 2, \dots \rangle, \dots$, where *n* is any natural number, can be put in one-one correspondence with the natural numbers.)

What of the first premiss? I confess I see little force in it. It decrees that progress in the performance of a sequence of tasks must consist in the performance of a successively larger fraction of those tasks. If that decree is accepted, then many performances will exhibit no progress. If I design to break every window in the quadrangle, I need not throw successive stones and do my job piecemeal; a single bomb, suitably placed, will blow in all the windows at once. And in any case, it is unnecessary to accept the decree. I may surely make progress in my plan to defenestrate the quadrangle, even if the actual blowing in of the windows occurs all at an instant: my progress might well be measured by the length of time still to elapse before my object is attained. Similarly, we may say (if we wish) that *b* is progressing in his task of touching all the a_i s, on the grounds that the time at which his task will be completed is getting nearer. In short, performance does not evidently require progress; and the notion of progress is in any case readily accommodated to apply to the performance of infinite tasks.

continuous, there are points t_1, t_2, \dots between t' and t (in fact there are infinitely many such points). At t_1 a is not ϕ , for t_1 is later than t' ; and at t_1 a is not non- ϕ , for t_1 is prior to t . But that is impossible. Hence, in general, if there is a first point of a 's non- ϕ ness there is no last point of a 's ϕ ness.-An exactly analogous argument shows that if there is a last point of a 's ϕ ness there is no first point of a 's non- ϕ ness.

The application to Zeno is plain. There is a first point of b 's having completed the tasks in S -viz. the point at which he touches B . Hence there is no last point at which he completes the tasks in S ; i.e., there is no last instant of his performance. That is no paradox: it simply brings out a general feature of all change: change can occur without there being a last moment of the unchanged state.

Thus the illustration of the lamp does show something about the logic of infinity; but it does not show that (2) is true. And I turn to the seventh and final argument for (2). The situation in which a performer of infinitely many tasks finds himself can be represented as a progress along an asymptotic curve:



In the Zenonian case, the curve plots b 's distance from B , the x -axis representing that distance, the y -axis marking off the a_i s. The curve constantly approaches 0 but never reaches it. In general, progress *via* an infinite sequence of operations can be represented by an asymptotic line on a graph; and the discontinuity or gap between the line and the y -axis provides a puzzle; for it appears to mark a gap in the causal nexus of events. The runner's progress through the a_i s is surely causally connected to his arrival at B ; yet there is an unbridgeable gap between his arrival at the successive a_i s and his arrival at B .

Thus, first, there is always a physical gap between b 's position at the end of any run to an a_i and his goal. How can causation send its spark across that spatial chasm? Second, there is no event among the touchings of the a_i s, which may be linked to the arrival at B ; for any causal chain would bind the arrival to the *last* touching, and there is no last touching. How can a causal chain hold firm when there is no link locked to its last link? Third, consider the period T during which b travels from A to B . There is no last instant of b 's travelling, and hence no last instant in T . Let t be the first instant after T : given the details of b 's progress we can say for any instant *within* T exactly where b

will be: at t_i he will be at a_i . Yet we cannot in the same way predict where b will be at t . (Reconsider the lamp: the plan to switch it on at 12.30, off at 12.45, and so on enables us to predict the lamp's state at any time *between* 12.00 and 1.00; it does not license a prediction for 1.00.) How can a causal law cover that hole in the path of predictability?

Thus we have a seventh reason for upholding (2): 'If (2) were false, the great chains of causation would snap'.¹¹

The argument deserves a longer presentation than I can give it here. I content myself with three summary observations. First, the argument does not show the truth of (2); at most it shows that (2) is true in a Laplacean world, where every event is causally determined by its predecessors; and no one has yet shown that our familiar world is Laplacean. Perhaps there are causal hiatuses; many philosophers and scientists hold that there are.

Second, the argument is stated too grossly. In the case of *some* infinite performances there seems no difficulty in formulating causal laws that 'bridge the gap'. The Zenonian runner provides such an example: given that he starts from A and proceeds at uniform speed through all the a_i s, we can predict that he will be at B at a given time after his departure from A . The fact that the a_i s are infinite and the distinct fact that no a_i is contiguous to B have no bearing on the question: causal 'chains'—a poor metaphor at the best of times—do not have to link events to their immediate neighbours; they can bind events together even if immediate neighbours are lacking.

Third, it is by no means clear that in *any* infinite performances there will or must be causal 'gaps'. Causal laws are empirical hypotheses; as far as I can see, nothing in the description of the infinite switchings of the lamp, or in any other infinite performance, rules out any causal hypothesis. Switch the lamp off at t_1 , on at t_2 ; and so on. Is the state of the lamp at t predictable? Is it causally determined by its earlier states? Those questions cannot be answered *a priori*. Suppose that numerous experiments of such switchings were carried out, and that in all of them the lamp was found to be on at B ; then we should have reason to favour the causal hypothesis that the lamp's state, after infinite switchings of the sort described, was *on*. Of course, a different result might be obtained: we might always find the lamp *off*; or there might be a random selection of *offs* and *ons*. Of course, we do not know, now, what the state at B would be. Of course, we shall never be able to conduct any such tests as those I have imagined. But those points are irrelevant: the seventh argument purports to show that only (2) will shield us from a world of random happenings. That is not so; and the notion of causation does not, I think, help us to ground our desire to believe (2).

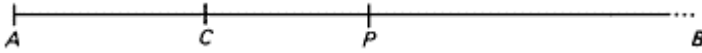
I believe (2) to be false: of the many arguments designed to support (2), all are wanting in one or more particulars. But I cannot show that (2) is false; indeed, the reason why Zeno's Dichotomy is so fascinating an argument is to be sought in (2): men want to believe (2); they cannot believe that we possess infinite powers; and they keep producing ever more ingenious arguments in favour of Zeno. For all that, until a new batch of arguments comes forward I shall continue to reject Zeno's conclusion by rejecting (2).

(c) *Achilles and the tortoise*

The second paradox in Aristotle's list is the Achilles:

Second is the one called Achilles. This says that the slow will never be caught in running by the fastest. For the pursuer must first get to where the pursued started from, so that it is necessary that the slower should always be some distance ahead (**186: Phys 239b14–8 =A26=26 L**).

Take a racecourse, AB , of indefinitely great length. Let Achilles be placed at A ; let the tortoise—as we have come to know his opponent—be placed at any point C between A and B ; and at t let Achilles and the tortoise each begin to move towards B . (The paradox says—and need say—nothing about their relative speeds, about the absolute speed of either, or about the uniformity of either's progress.) Suppose that Achilles does catch up with the tortoise; i.e., that there is some point P on AB such that at some time t' after t both Achilles and the tortoise are at P . Since the tortoise has been moving towards B , P is between C and B , thus:



It follows that 'the pursuer must first get to where the pursued started from'; i.e., that at some time between t and t' Achilles is at C . Now when Achilles is at C , the tortoise is at some point, C_1 , between C and B ; but clearly Achilles must reach C_1 before t' ; and when he is at C_1 , the tortoise is already ahead, at C_2 , between C_1 and B . And in general, if Achilles is at C_i , the tortoise is already at C_{i+1} , one step ahead of him. Thus 'it is necessary that the slower should always be some distance ahead', and Achilles can never catch the tortoise.

According to Aristotle, the Achilles paradox is merely a twopenny coloured version of the Dichotomy. Achilles is dramatized (*tetragôdounenos*), but at bottom 'this argument is the same as the Dichotomy (only differing in that the added magnitude is not divided in half)...so that it necessarily has the same solution' (*Phys 239b18–20; 25–6*). Most modern scholars disagree with Aristotle; at all events, the Achilles is regularly discussed at length and in its own right, whereas the Dichotomy is regularly ignored. (Most of the arguments about infinite performances which I have examined under the rubric of the Dichotomy were originally advanced in connexion with the Achilles.) And at first sight, things do seem to go against Aristotle: according to the Dichotomy, Achilles can never reach the tortoise's starting position; according to the Achilles, even if he could do so, he could never catch the tortoise; in the Dichotomy, the race has a fixed finishing post, which can never be reached; in the Achilles, the finishing post itself is perpetually receding. Achilles, it seems, has double the toil and trouble of his undramatic counterpart in the Dichotomy. He cannot move; and if, *per impossibile*, he could, he would never reach his goal.

Nevertheless, a closer inspection of the Achilles vindicates Aristotle's judgment: the paradox, if not identical with the Dichotomy, is no more than the Dichotomy with an unharmonious coda.

The last step of the argument is invalid. From:

(1) For every i , if Achilles is at C_i , then the tortoise is at C_{i+1} , Zeno invites us, in effect, to infer:

(2) For every point p on AB , if Achilles is at p , the tortoise is at some point p' between p and B .

Now (1) is true; and Zeno's argument shows it to be true. And (2) does imply that Achilles never catches the tortoise. But (2) does not follow from (1). (2) could be inferred from the conjunction of (1) and:

(3) For every point p on AB , there is an i such that C_i is between p and B .

But clearly (3) is not available to Zeno. Every C_i is, by construction, between C and the hypothetical meeting point P ; for C_i is simply the point on AB where the tortoise is at the time when Achilles is at C_{i-1} . Thus on the twin assumptions that the tortoise never ceases to move and that Achilles' speed is finite, for no i does $C_i=C_{i-1}$. But if $C_i=P$, then $C_i=C_{i-1}$; hence for no i does $C_i=P$. Evidently, no C_i is between P and B . Hence every C_i is between C and P . It follows that Achilles and the tortoise never meet at any C_i , as (1) says. But that conclusion is now seen to be the merest triviality: the two runners will never meet at any point before their first meeting point. From this, nothing like (2) follows; from the fact that they do not meet before they meet, we can scarcely infer that they never meet.

Aristotle saw all that very clearly: 'when [the tortoise] is ahead, he is not caught; but nevertheless he is caught if you grant that one can traverse a limited distance' (*Phys* 239b27–9). According to Aristotle, (1) is true but trivial. How, then, can we get any paradox out of the Achilles? Only, Aristotle implies, by denying that Achilles can traverse the finite distance AP . And how might Zeno propel us towards such a denial? Only, Aristotle implies, by adducing the considerations that he advanced in the Dichotomy. Thus (1) brings out, in a clear enough fashion, the fact that before Achilles reaches the tortoise he must touch infinitely many points; and it is easy to see from (1) that however close to the tortoise Achilles may be—however many C_i s he may have successively touched—he still has infinitely many C_i s still to touch before he reaches P . And such infinite performances are, Zeno invites us to suppose, impossible.

That manoeuvre reduces the Achilles to the Dichotomy; and any objections to the argumentation in the latter paradox will apply immediately to the former. I do not deny that the Achilles is both clever and elegant; but I agree with Aristotle that it raises no philosophical difficulties which its more prosaic predecessor has not already flushed out.

(d) *The arrow*

The third paradox of motion does break new ground. It is the Arrow. Aristotle's brief description and curt dismissal read thus:

Third is the one we have just mentioned—that the travelling arrow is at rest. This comes about from assuming that time is composed of ‘nows’ (*ta nun*); for if that is not granted, there will be no deduction (**187: *Phys* 239b30–3=A 27=28 L**).

Aristotle refers back to the opening of Z 9, where the transmitted text reads as follows:

Zeno misargues; for if, he says, everything always rests or moves whenever it is against what is equal (*kata to ison*), and what is travelling is always in the now (*en tōi nun*), the travelling arrow is motionless (**188:239b5–7=A 27=29 L**).

After that brief report there is the same dismissal as at 239b30–3.

Two further, non-Aristotelian, texts may be adduced. Epiphanius gives the following report:

He [sc. Zeno] also argues thus: what is moving moves either in the place in which it is or in the place in which it is not. And it moves neither in the place in which it is nor in that in which it is not. Therefore nothing moves (**189: *advenus haereticos* III. 11=18 L**).

The second text, from Diogenes, repeats the argument in an abbreviated form (**IX.72=B 4=17 L**). Some scholars think that this dilemma is an independent Zenonian argument, the fifth paradox of motion; others, judging it feeble in itself, attach it to the Arrow: ‘Either the arrow moves in the place where it is, or it moves in the place where it is not; evidently it cannot move in the place where it is not; by the Aristotelian argument it cannot move in the place where it is: hence it cannot move.’ That reconstruction may possibly be right; but it has no ancient warrant. Moreover, the sources who ascribe the dilemma to Zeno are not unimpeachable; and elsewhere the dilemma is associated with the name of Diodorus Cronos.¹² I incline to the sceptical view that a later dilemma has been anachronistically fathered on Zeno. However that may be, I do not think that the dilemma is very exciting; and I shall confine my discussion to Aristotle’s text.

That text is hard enough, in all conscience. It offers Zeno two premisses:

- (1) If *a* is ‘against what is equal’ at *t*, then either *a* rests at *t* or *a* is moving at *t*.
- (2) If *a* is moving at *t*, then *a* is ‘in the now’ at *t*.

From these propositions we are invited to conclude that:

- (3) If *a* is moving at *t*, *a* is not moving at *t*.

And hence:

- (4) *a* is not moving at *t*.

(I have replaced Aristotle’s word ‘travelling’ (*pheromenon*) by ‘moving’ in (1) and (3); the terms are synonymous.)

The inference from (3) to (4) is valid; and it is worth noting that Zeno uses a subtle theorem of propositional logic later known as the *Lex Clavia*: if if *P* then not-*P*, then not-*P*. (The companion law, the *Consequently Mirabilis*—if if not-*P* then *P*, then *P*—was used in a celebrated context by Aristotle.) The move from (3) to (4) is, however, the only uncontroversial element in the paradox: all scholars recognize the obscurity in

(1) and (2) and the difficulty in moving from them to (3); and many philosophers deny that (4) is sufficient to establish Zeno's desired conclusion, that nothing moves. I shall first deal with (1) and (2).

(1) and (2) will not yield (3). Instead of (1) and (2) we might well expect:

(5) If *a* is 'against what is equal' at *t*, then *a* rests at *t*.

(6) If *a* is moving at *t*, then *a* is 'against what is equal' at *t*.

If we add the further premiss:

(7) If *a* rests at *t*, then *a* is not moving at *t*, we can validly infer (3). Premiss (7), which may well seem a thoroughly trifling proposition, is easy enough to supply; but (5) and (6), neither of which follows from (1) and (2), surely need explicit statement. Scholars have accordingly emended the text of the *Physics* so as to produce (5) and (6).

The simplest means of producing (5) is the excision of the words 'or moves' (*ê kineitai*) from 239b6; then (1) vanishes from the argument and is replaced by an explicit statement of (5). (6) might be derived from (2) in conjunction with:

(8) If *a* is 'in the now' at *t*, then *a* is 'against what is equal' at *t*. It is possible to introduce (8) into Aristotle's text; but it is perhaps preferable to treat (8) as a suppressed premiss. Zeno states (2) and expects us to infer (6) by way of (8).¹³

Thus I suggest that Aristotle's text originally presented (5) and (2) explicitly, and expressly inferred (3) from those two premisses. And I suggest that Zeno's original argument started from (5), (2) and (8); that it first inferred (6) from (2) and (8); that it then inferred (3) from (5), (6) and (7); and that it finally inferred (4) from (3), concluding that nothing moves. The textual suggestions are, I think, of no great importance: what matters is that Zeno intended us to reach (3), and hence (4), from (5) and (6).

What, then, is the precise sense of Zeno's premisses? In particular, how are we to understand the two odd phrases 'in the now' and 'against what is equal'? 'The now (*to nun*)' is Aristotle's standard term for an instant of time; but it makes no sense to say, baldly, '*a* is at *t*' where *t* names some instant: I may be in a place, but I cannot simply be 'at a time'. We might construe 'is (*esti*)' strongly, as 'exists'; or, better, we might take it as a verb-variable, so that '*a* is in the now' means '*a* ϕ s at *t*'—for some suitable verb ϕ and some definite instant *t*. (Aristotle says '*in the now (en tōi nun)*'; I say '*at t*': some philosophers find a significant difference here; but I think that 'at' is simply the appropriate English translation of '*en*'.)

The commentators all gloss 'is against what is equal' by 'occupies an equal space'; and the gloss is surely correct. Most of them add that 'an equal space' is elliptical for 'a space equal to its volume'; so that '*a* is against what is equal' becomes '*a* occupies a space equal to its own volume':

No Creature loves an empty space;
Their Bodies measure out their Place.

I accept that explanation: nothing else will give Zeno an argument of any plausibility.¹⁴

Thus interpreted, are Zeno's premisses true? Premiss (2) has become:

(2*) If *a* is moving at *t*, then there is some instant *t'* such that at *t a* is ϕ ing at *t'*.

And that is a tautology. The suppressed premiss (8) reads:

(8*) If there is some instant t' such that at t a is ϕ ing at t' , then at t a occupies a space equal to its own volume.

That is a peculiar observation; and I cannot divine why Zeno should have propounded it. Indeed, the very notion of being 'in the now' seems only to add an unnecessary complication to Zeno's argument; for (6), which has now become:

(6*) If a is moving at t , then at t a occupies a space equal to its own volume,

can be justified on far less mysterious grounds: it is a necessary truth that everything always occupies a space exactly equal to its own volume; i.e., that for any object x and time t , the volume of space occupied by x at t is equal to the volume of x at t . For what is the volume of an object if not the amount of space it occupies? But then whether or not a is moving at t , a is 'against what is equal' at t ; hence (6) is necessarily true.

What, then, of premiss (5)? If that is true, then Zeno has reached (4) and we are potentially in trouble. (5) now reads:

(5*) If a occupies at t a space equal to its own volume, then a rests at t .

Some philosophers hold that (5*) is not even coherent, let alone true. According to Aristotle, 'at an instant, it is not possible for anything to be either in motion or at rest' (*Phys* 239bl; cf. 234a32). Rest is a matter of endurance: things rest *for a period* of time; they cannot intelligibly be said to rest *at a point* in time. Aristotle has a multitude of followers. The question is usually discussed in connexion with Zeno's conclusion, (4); and I shall follow the custom. For at step (5) Zeno is easily and trivially defended; we need only replace (5*) by:

(5⁺) If a occupies at t a space equal to its own volume, then it is not the case that a is moving at t .

Indeed, (5⁺) not only evades the captious objection about rest; it also enables Zeno to dispense with the additional premiss (7): (5⁺) and

(6*) together entail (3) with no more ado.

Those many thinkers who agree with Aristotle that 'at an instant, nothing moves' are obliged to accept (5⁺). And (6*) is certainly true. Since (5⁺) and (6*) entail (3), it seems that Zeno is vindicated: the moving arrow does not move.

(e) *Movement in a moment*

Russell, for one, happily assents to (4): Zeno, he says, did prove that 'we live in an unchanging world, and that the arrow, at every moment of its flight, is truly at rest. The only point where Zeno probably erred was in inferring (if he did infer) that, because there is no change, the world must be in the same state at one time as at another.' In short, (4), far from being a monstrous paradox, is 'a very plain statement of a very elementary fact'. But (4) does not entail the absence of motion. Rather, it enables us to see more clearly the real nature of motion: 'People used to think that when a thing changes, it must be in a state of change, and that when a thing moves, it is in a state of motion. This is now known to be a mistake. When a body moves, all that can be said is that it is in one place at one time and in another at another.'¹⁵

Russell's views are not entirely plain. I take it that he is saying three things about the inference from (4) to the denial of motion—strictly speaking, about the inference from:

(9) ($\forall t$) (if t is in T , then a is not moving at t),

to:

(10) *a* does not move during *T*. Russell says *first* that the inference is invalid; *second*, that a correct understanding of the concept of motion will reveal its invalidity; and *third*, that Zeno may well not have made or intended the inference. I find Russell's view bizarre; on all three counts his reaction to the Arrow is mistaken. I take the points in reverse order.

First, then, all the ancient commentators treat the Arrow as part of Zeno's general attack upon motion. They plainly regard (4) as the penultimate step leading to a negation of motion: I see no reason to dispute their view; and I suppose that Zeno ended his argument by explicitly saying 'the arrow does not move'—indeed it is natural to construe Aristotle's words at 239b7 in precisely that way.

Second, let us consider Russell's account of motion. Russell naughtily describes it as a 'static' account; it can, I think, be formulated as follows:

(D1) *a* moves during *T* if and only if for every pair of distinct instants t_1 and t_2 in *T* there is an instant t_3 between t_1 and t_2 and a pair of distinct places p_1 and p_3 such as *a* is at p_1 at t_1 and *a* is at p_3 at t_3 .

That looks needlessly complicated. In fact its complexities are necessary, and must be multiplied (as the ancients realized) if we are to say of spinning tops that they move. Zeno in (4) talks of motion at an instant; does (D1) show such talk to be odd? does it show the inference from (9) to (10) to be invalid? Motion at an instant is easily defined:

(D2) *a* is moving at *t* if and only if for some *T* *t* is within *T* and *a* moves during *T*.

Given (D2), (9) immediately implies (10): if (D1) and (D2) offer a correct account of motion, they do not thereby reveal the invalidity of the Zenonian inference. Quite the contrary. (I do not mean to imply that Zeno himself had in mind a Russellian account of motion. Nor do I think that there are just two ways of envisaging motion: the one given in (D1–2), and the one rightly rejected by Russell according to which motion is an intrinsic quality of the moving object, in much the way that triangularity is an intrinsic quality of a triangular object. My point is only that Zeno has nothing to fear from (D1).)

Finally, is the inference from (9) and (10), or from (4) to Zeno's immobile conclusion, valid? It seems to me gratuitously paradoxical to deny its validity: if the inference is invalid then we must, in Bergson's celebrated phrase, accept 'the absurd proposition that movement is made of immobilities'.¹⁶ Every period of motion will consist of an infinite sequence of motionless states. Ordinary usage accustoms us to talk of 'motion at an instant': I can say that the car was travelling at 34 m.p.h. at the moment when the bus hit it; or that at 9.10 this morning I was cycling into College. And ordinary usage sanctions inferences of the relevant sort between statements of 'motion at an instant' and statements of enduring motion: if I claim to have been cycling between 9–05 and 9.15 I cannot consistently deny that I was cycling at 9.10; if the car moved steadily at 34 m.p.h. until it hit the bus, then it was moving at 34 m.p.h. when it hit the bus. Indeed, it is, I think, a general truth that if $a \phi$ s during *T*, then *a* is ϕ ing at every instant *t* in *T*; and the acceptability of that proposition in no way depends on restricting the range of ' ϕ ' to static verbs.

Why should any philosopher go against that natural mode of speech and argument? It cannot be said that the notion of 'motion at an instant' makes no sense; clearly it does make perfectly ordinary sense, and a sense that can be lucidly articulated in some such

definition as (D2). Are there, then, hidden inconsistencies in the definition, or any that may replace it?

Some have argued as follows: ‘All motion has duration, and it always makes sense to ask, of a moving object, how long it has moved for; consequently, the notion of instantaneous motion is logically contradictory; and thus motion at an instant is impossible.’¹⁷ It is true that all motion involves duration: if *a* moves, then there is some period *T* such that *a* moves during *T*. And it does follow from this that instantaneous motion is a logical impossibility: ‘*a* ϕ s instantaneously’ means, I take it, that for some *t* *a* ϕ s at *t* and for no *T* does *a* ϕ during *T*. In that sense some things, e.g., dying or learning—may or may not be instantaneous; and some things, of which change and all its species are the most conspicuous examples, are necessarily not instantaneous. But that does not show that the argument I have just considered is sound; for its final step involves a gross confusion: it is one thing to reject instantaneous motion, another to reject motion ‘at an instant’. To believe that motion at an instant is possible is to believe that ‘*a* moves at *t*’ is consistent; to believe that instantaneous motion is impossible is to believe that ‘*a* moves at *t* and for no *T* does *a* move during *T*’ is inconsistent. Plainly, those two beliefs are compatible: I uphold motion ‘at an instant’; but I also believe that if *a* moves then for some *T* *a* moves during *T*, and thus I reject instantaneous motion.

Aristotle provides a different argument against motion at an instant: ‘That nothing moves at an instant is evident thus: if it did, a thing could move both quicker and slower. Let *N* be an instant, and let the faster thing have moved the distance *AB* at *N* (*en autôî*). Now at the same instant (*en tôî autôî*) the slower thing will have moved a shorter distance, say *AC*. But since the slower has moved through *AC* in the whole instant, the faster will have moved [through *AC*] in a shorter time than this—so that the instant will have been divided. But that is impossible—hence it is not possible to move at an instant’ (*Phys* 234a23–31).

There are two related errors in this argument. The first is more evident in the translation than in the Greek: it consists in treating the term ‘instant (*nun*)’ as though it connoted a period of time, as though it were the last member of the set of terms that includes ‘year’, ‘day’, ‘hour’, ‘minute’, ‘second’. In English we do sometimes talk of things happening ‘in an instant’, and the same idiom is, I suppose, possible in Greek. But when we say ‘in an instant’ we are using the term ‘instant’ loosely, as a synonym for ‘split second’. Aristotle misses this point; and his argument depends on his construing ‘*en tôî nun*’ as though it were logically on a par with ‘*en mi ai hôral*’ (‘in an hour’).

The second, allied, error, is harder to exhibit clearly. It lies in Aristotle’s supposition that ‘the faster thing has moved the distance *AB en autôî*’. Behind that supposition lies the truism that if *a* has moved at all, then there is some distance through which *a* has moved. But from that truth Aristotle falsely infers that if *a* moved at *t*, then there is some distance through which *a* moved at *t*, say the distance *AB*. Now he might properly have inferred that there is some distance through which *a* was moving at *t*: at *t* *a* was moving from *A* to *B* if *t* falls within a period of constant motion *T* and if during *T* *a* moved from *A* to *B*. But from ‘*a* was moving from *A* to *B* at *t*’ we cannot infer ‘*a* moved from *A* to *B* at *t*’. Indeed, we cannot, in general, infer ‘*a* ϕ ed at *t*’ from ‘*a* was ϕ ing at *t*’ (as, in another context, Aristotle in effect recognizes). I was eating breakfast at 8.45, but I did not eat breakfast at 8.45; I was reading a book at 10.30, but I did not read a book at

10.30. In general, if ' $a \phi$'s' entails 'for some T , $a \phi$'s during T ', then ' a is ϕ 'ing at t ' does not entail ' $a \phi$'s at t '.

Thus Aristotle's argument against motion 'at an instant' fails. And I know of no other, more compelling, argument.

I end this section by considering Aristotle's objection to the Arrow: he says simply that 'time is not composed of indivisible news' (239b8). I take it that he means to ascribe the following argument to Zeno:

(11) $(\forall t)$ (if t is in T , a is not moving at t).

(12) T is composed of the set of instants it contains.

Hence:

(13) a does not move during T .

Here (11) represents (4), and (13) is a version of Zeno's wholesale rejection of locomotion. And Aristotle objects that (12) is false.

I have two comments. First, we might disagree with Aristotle about (12) and yet agree with him that the argument from (11) to (13) is unsound. For the pattern of inference represented by (11)-(13) is invalid: it is a case of the 'fallacy of composition', the fallacy of arguing from 'All the parts of X are F to ' X is F '. (All the molecules that make up this glass of beer are tasteless, so the beer is tasteless.) Second, I see no reason to ascribe Aristotle's argument to Zeno: Zeno infers (13) directly from (11). There is no need to invoke (12); and hence no occasion to charge Zeno with a fallacy of composition.

(f) *The arrow blunted*

If I am right, we cannot allow Zeno to reach proposition (4) without giving him his conclusion that motion is impossible. Since (6*) is true, we must reject (5⁺); and indeed (5⁺) seems to me to be clearly false: objects do, at every instant in their temporal careers, occupy a space exactly equal to their volume at that instant. And they do so even if they are in motion throughout their temporal careers. Why should anyone find that puzzling?

Some might argue as follows: 'If we think of the arrow as occupying a given position for a time of zero duration, it will be obvious enough that it cannot be moving *then*: it will have no time in which to move.'¹⁸ That is a bad argument: a needs time to move from A to B ; but a needs no time *to be moving* from A to B . The arrow is moving at t ; it does not follow, as I have already said, that the arrow *moves* through some distance at t . If that argument lies behind (5⁺), then (5⁺) is not established.

But the argument makes no essential reference to the space which a occupies at t ; and I suspect that a different consideration operated on Zeno's mind. I suspect that he argued as follows: 'If at t a occupies a space no greater than itself, then a has no room in which to move. Moving involves the transition from one place to another, and hence occupancy of more than one place. But at t a occupies just one place, viz. the place marked out by its own boundary. That argument too is bad; and the grounds of its badness have already been presented: if a moves, then there are two points, A and B , such that a moves from A to B ; and if a is moving at t , there are points A and B such that at t a is moving from A to B . But it does not follow that if a is moving at t , then

there are points A and B such that at t a moves from A to B . Similarly if I smoke my pipe, there is a plug of tobacco which I consume in the process; and if I am smoking a pipe at t , then there is a plug of tobacco which I am consuming at t . But it does not follow—and it is not true—that if I am smoking a pipe at t , then there is a plug of tobacco which I consume at t .

It is perhaps worth setting out in one paragraph the various facts about motion which I have tried to express and to deploy. First, motion requires duration: ' a moves' entails 'For some period T , a moves during T '. Second, motion requires extension: ' a moves' entails 'For some distance AB , a moves from A to B .' Third, motion 'at an instant' requires durational motion: ' a is moving at t ' entails 'For some period T , a moves during T '. Fourth, motion 'at an instant' requires motion through a space: ' a is moving at t ' entails 'For some distance AB , a moves from A to B '. Fifth, motion 'at an instant' does *not* require instantaneous motion: ' a is moving at t ' does not entail 'For no period T , a moves during T '. Sixth, motion 'at an instant' does *not* require instantaneous transition: ' a is moving at t ' does not entail 'For some distance AB , a moves from A to B at t '. There is evidently much more to be said about the logic of motion than that. But I believe that the six facts I have just listed are sufficient to show that the customary answers to Zeno's Arrow are mistaken. Zeno's argument is valid; but it relies on a false premiss.

There remains the possibility of an eleventh hour reprieve: so far I have taken Aristotle's phrase 'the now' to denote an instant; and one of my complaints has been against the misconstrual of instants as small periods of time. Now some scholars suppose that Zeno's argument should in fact be conducted entirely in terms of periods of time, and not at all in terms of instants. I do not think that Aristotle's text encourages such a supposition; and I do not believe that we can seriously hope to recover a genuinely Zenonian argument behind a distorted Aristotelian presentation. Nonetheless, it is clearly desirable to see what difference might be made to the penetrative powers of the Arrow if it is freed from the notion of movement 'at an instant'.

In the new version proposition (5) is replaced by:

(14) If a occupies during T a space equal to its own volume, then a is at rest during T .

As it stands, (14) seems plausible, or at least more plausible than (5). But in fact it is ambiguous. Its plausible sense is more explicitly given by:

(14a) If there is a place p equal to the volume of a such that at every instant t in T a occupies p , then a is at rest during T .

Indeed, (14a) is not only plausible: it is evidently true. Clearly, however, (14a) and (6) do not together entail (4). To secure the entailment, (6) too must be replaced by a proposition talking about periods of time. The proposition required is:

(15) If a is moving during T , then there is a place p equal to the volume of a such that at every instant t in T a occupies p . And just as (14a) is evidently true, so (15) is evidently false.

If (14) is read as (14a), the Arrow has no penetration. The alternative reading of (14) is:

(14b) If at every instant t in T there is a place p such that a occupies p at t , then a is at rest during T .

A suitable replacement can be found for (6) which, together with (14b), entails (4). And that replacement will be true. But now (14b) turns out to be false; for the antecedent of (14b) is compatible with there being distinct places $p_1, p_2 \dots$ which a occupies at distinct instants $t_1, t_2 \dots$ in T ; and occupancy of distinct places during T is incompatible with rest during T . Thus if (14) is interpreted as (14b) the Arrow gets no further. And I conclude that the proposal to read Zeno's paradox in terms of periods of time rather than in terms of instants has no philosophical merit. Not only does it fail to give Zeno a better argument; it also fails to raise any interesting philosophical puzzles.

Let me end my discussion of the Arrow with two negatively polemical remarks. It is usually supposed that Zeno's paradox carries with it some philosophical theory about the nature of time; and Zeno's commentators regularly adduce rival theories in the course of their reflexions upon it. My discussion has shown the falsity of that common supposition; for in expounding and criticizing the Arrow I have neither explicitly nor implicitly invoked any theory of time. In particular, I have not accused Zeno of treating time as being 'composed of instants'; nor have I ascribed to him the view that time is made up of 'atomic minimal parts'; nor have I made him assume that motion proceeds cinematographically. The paradox, as we should expect and desire, is innocent of any such theories: it presupposes only the two harmless and common notions that there are instants, as well as periods, of time; and that things move, if at all, at instants.

Many scholars find an architectonic structure uniting Zeno's four surviving paradoxes of motion: the paradoxes form a dilemmatic attack upon movement, the Dichotomy and the Achilles supposing that time and space are infinitely divisible, the Arrow and the Stadium supposing that there are indivisible spatial and temporal *quanta*. My account of the Arrow seems to me to have revealed the falsity of that neat fantasy.

(g) *The stadium*

The fourth and final paradox is the Stadium:

[i] The fourth is the one about equal bodies which move alongside equal bodies in the stadium from opposite directions—the ones from the end of the stadium, the others from the middle—at equal speeds, in which he thinks it follows that half the time is equal to its double, [ii] The misargument consists in requiring that [a body travelling] at an equal speed travels for an equal time past a moving body and a body of the same magnitude at rest. That is false, [iii] E.g., let the stationary equal bodies be AA ; let BB be those starting from the middle of the As (equal in number and in magnitude to them); and let CC be those starting from the end (equal in number and in magnitude to them, and equal in speed to the Bs). [iv] Now it follows that the first B and the first C are at the end at the same time, as they are moving past one another, [v] And it follows that the C has passed all the As , and the B half; [vi] so that the time is half, [vii] for each of the two is alongside each for an equal time, [viii] And at the same time it follows that the first B has passed all the Cs . [ix] For at the same time the first B and the first C will be at opposite

ends, [x] being an equal time alongside each of the *Bs* as alongside each of the *As*, as he says, [xi] because both are an equal time alongside the *As*. [xii] This is the argument, and it rests on the stated falsity (190: *Phys.* 239b33–240a18=A 28=35 L).

The outline of that argument is clear enough. We have three equinumerous groups of bodies in a stadium, and the bodies are all equal in size. How many bodies each group possesses is not stated: the number is immaterial to the argument; and I shall take the simplest case, that in which each group contains just two bodies.¹⁹ Aristotle does not say that the members of each group are contiguous and arranged in a row (like the carriages of a railway train); but the argument plainly requires that assumption. One row of bodies, the *As*, is stationary. The *Bs* and the *Cs* move; and I shall refer to the leading *B* and the leading *C* as *B*₁ and *C*₁.

Aristotle gives, in [i] and [iii], a description of the stadium before the *Bs* and *Cs* begin their movements. I shall call this the *starting position*. Then [iv]–[x] list three things that ‘follow’ once the moving bodies have left the starting position. Let us call the starting time *t*₁; then at some later point, *t*₂, the bodies will be in what I shall call the *crucial position*. And if we consider what has happened during *T*, the interval between *t*₁ and *t*₂, we shall (according to Zeno), be faced with paradoxical or contradictory results. The problems of interpretation are thus three: first, to determine the starting position, at *t*₁; second, to determine the crucial position, at *t*₂; third, to discover what paradox is supposed to arise from our consideration of *T*, and how it is supposed to arise. The problems cannot be tackled separately; but it helps, I think, to begin by looking at the second of the three things that allegedly ‘follow’.

That second result is stated in [v]–[vii], which describe, or partially describe, the state of the stadium at the crucial time, *t*₂. The description contains a vagueness; but I shall simply assume that the phrases ‘the *B*’ and ‘the *C*’ in sentence [v] denote the *first B* and the *first C*. Thus sentence [v] asserts that at *t*₂ the first *C* has passed all the *As* and the first *B* has passed half the *As*; *C*₁ has passed two *As*, *B*₁ has passed one *A*. Given that the *Cs* and the *Bs* are proceeding at the same speed, we can infer that at some time prior to *t*₂—call it, for the moment, *t*_{*x*}—the stadium looked like this:

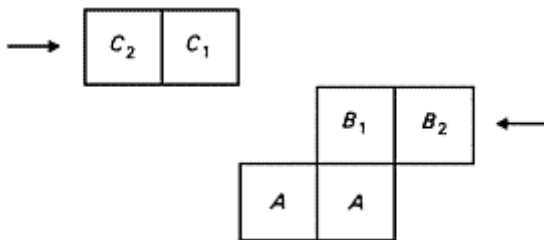


fig.1

As far as *t*₂ goes, each of the following two diagrams seems to be consistent with the words of Aristotle’s description:

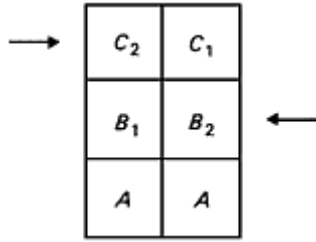


fig 2.1

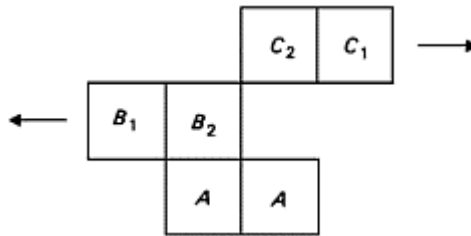


fig 2.2

Now Zeno infers in [vi] that ‘the time is half’. The obvious expansion of this reads: ‘the time taken by B_1 to get from its *fig. 1* position to its new position is half that taken by C_1 to get from its *fig. 1* position to its new position’. And Zeno’s argument for that, in [vii], glosses as follows: ‘In the interval between *fig. 1* and the new position, B_1 has passed half as many A s as C_1 has; and both B_1 and C_1 are travelling at the same speed.’ That reasoning allows us to fix on *fig. 2.1* as the correct diagram for t_2 .

Let us now glance briefly at the *first* result, stated in [iv]. I assume that the time of this result is the same as the time of the second result, i.e. t_2 . The run of the text favours the assumption; and if we introduce a further time I see no way of reconstructing any argument. At t_2 , then, B_1 and C_1 are ‘at the end’. That phrase is wholly opaque. Four glosses at least are possible; and three of those glosses produce a plurality of diagrams. But given that *fig. 2.1* represents the stadium at t_2 , we can limit the choice to two diagrams, (a) If ‘at the end’ means ‘at the [opposite] end[s] of the A s’, then the diagram must be *fig. 2.1* itself, (b) If ‘at the end’ means ‘[each] at the end [of the other’s row]’, the diagram must be:



fig. 3

One manuscript has a reading which shows that *fig. 3* represents an ancient interpretation; and since we do not want the second result merely to repeat the first, we should clearly plump for (b) and *fig. 3*.

We have thus identified the ‘crucial position’ as the position diagrammed in *fig. 2.1*, and it is an obvious conjecture that *fig. 1* represents the starting position at t_1 . Can that conjecture be supported from the text?

Aristotle describes the starting position twice, in [i] and in [iii]. The latter sentence adds something to the former: it specifies that the Bs start ‘from the middle of the As’. In *fig. 1* they do; and the Cs similarly start ‘from the end [of the As]’. That confirms that *fig. 1* represents the starting position of the three rows at t_1 .

There is an objection to my reconstruction. If the bodies start in the position depicted in *fig. 1* they will never reach the position of *fig. 2.1*; it is logically impossible, given the conditions Zeno stipulates, that both *fig. 1* and *fig. 2.1* should depict points in the careers of the Bs and Cs. But that objection is not fatal: Zeno, after all, is trying to find contradictions in the concept of motion; and we should not necessarily be dismayed to find that his paradoxes represent as achieved what in reality and logic is impossible. The question, then, is not: Can *fig. 1* yield *fig. 2.1*? It is: Can Zeno have thought, or have made it seem plausible to think, that *fig. 1* yields *fig. 2.1*? And we might attempt to answer that question by looking at the *third* result, sentences [viii]–[xi].

Textually, that too is uncertain. In [viii], at 240a13, where I translate ‘the first *B*’, most manuscripts read ‘*ta B*’, ‘the Bs’. Given that reading, we should have ‘at the same time’—i.e. at t_2 —the following position:

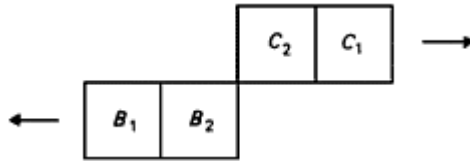


fig. 4

Now *fig. 4*, though a part of *fig. 2.2.*, is not reconcilable with *fig. 2.1*. Nor can I find any plausible argument which makes use of *fig. 4*. Consequently, the majority reading must be rejected; and we must read either ‘*to B* (the *B*)’ or ‘*to prōton B* (the first *B*)’.²⁰

We now have an argument designed to show that the third result actually occurs: i.e., that:

- (1) At t_2 B_1 has passed both C_1 and C_2 . The text presents three sentences: sentence [ix] gives:
- (2) At t_2 B_1 is against C_2 and C_1 is against B_2 . Sentence [x] gives:
- (3) C_1 is alongside each B for the same length of time that it is alongside each A . Sentence [xi] gives:
- (4) The Bs and the Cs spend an equal time alongside each A . Each of these three sentences allegedly supports its predecessor. Can anything be made of the argument?

Proposition (2) re-expresses the first result; and (1) follows easily enough from it and the initial description of the stadium. Proposition (4) is true, given the equal speed of the Cs and the Bs. Zeno, I suppose, argued thus: ‘If, as (4) says, B_1 spends n time units

against an *A*, and C_1 spends n time units against an *A*, then plainly B_1 spends n time units against C_1 . Thus (3) is established.²¹ And since by t_2 C_1 has been alongside *two* *As*, it must—by (3)—have been alongside *two* *Bs* as well. Hence (2). Finally, given the starting conditions, we may deduce (1).’

Let me now try to restate Zeno’s whole argument in briefer and more perspicuous fashion. Suppose that at t_1 the *As*, *Bs* and *Cs* are arranged as in *fig. 1*; and let the *Bs* and *Cs* move as specified. Let t_2 be the time at which C_1 has passed every *A*. Now C_1 has passed two *As* and hence two *Bs*; therefore *fig. 2.1* represents the position at t_2 . C_1 and B_1 each take n units of time to pass a body. Hence T , the period from t_1 to t_2 , is n units long. But in T B_1 has passed only one *A*. Hence $T=n$. Hence $T=1/2T$.²²

The argument is not sound; but no version of the Stadium will yield a sound argument. It is not elegant; but Aristotle’s text does not suggest an argument with the elegance of the Achilles. I am inclined to think that this reconstruction is as close to Zeno as anything we can now produce; and I have expounded it at some length because it goes against the prevailing orthodoxy.

Different interpretations fall into two classes. First, there are those which agree that *fig. 1* represents the starting position.²³ Such interpretations are only minor variants on mine; and they do not require special discussion. Second, there are those which abandon *fig. 1* and adopt as a starting position:

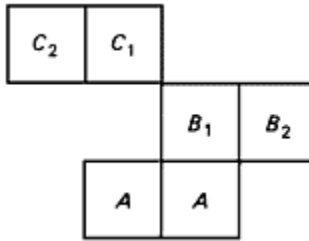


fig. 5

From *fig. 5* the position of *fig. 2.1* is easily reached; and in that lies the advantage of *fig. 5* over *fig. 1*. For my part, as I have already intimated, I do not think that the advantage is very great. And on the other side, adoption of *fig. 5* requires two changes in the text of **190**, one of which is most implausible.²⁴ But I shall not pursue the question further: philosophically speaking, it matters little whether we choose *fig. 1* or *fig. 5*.

(h) *A most ingenious paradox?*

Thus far I have concentrated on textual *minutiae* and little problems of interpretation; and it may well be wondered whether such attention is worth the paper it consumes. Aristotle dismisses Zeno’s argument curtly enough: Zeno, he says, wrongly assumes that the *Cs* will take as long to pass a moving *B* as they will to pass a stationary *A*. To most scholars the criticism seems both apt and conclusive: the paradox of the Stadium is philologically the most complicated of Zeno’s four arguments; philosophically it is the

simplest and the least interesting—correct a childish mistake and all is in motion again. As Eudemus long ago saw, the puzzle is ‘very silly, because the misargument is obvious’ (fr. 106W=A 28).

Moved by those considerations, yet convinced of Zeno’s logical acumen, some scholars have inferred that Aristotle has misrepresented Zeno. Zeno’s argument, they say, in fact assumed that the *As*, *Bs* and *Cs* were *minimal* bodies, indivisible atoms of stuff. And it further assumed that the time taken for a *B* or a *C* to pass a stationary *A* was a *minimal* period of time, an indivisible temporal *quantum*. Given those two assumptions, the Stadium becomes a sound and a significant argument; for in effect it shows that absurd results follow from such an atomic attitude to matter and time. The Stadium, in short, is an early essay in the logic of the continuum.²⁵

I do not deny the philosophical interest of that sophisticated version of the Stadium paradox; but it was not Zeno’s. There is no evidence that anyone prior to Zeno had entertained the atomistic theory he is imagined to be attacking; and there is no reason why he should himself have invented such a theory simply to knock it down. The sophisticated Stadium has no historical support. Nor is the train of reasoning which introduced it very compelling. It is simply false that Zeno was a brilliant logician who would never have committed an elementary error—I have already discussed more than one argument indubitably Zenonian and of less subtlety than the Aristotelian Stadium. Indeed, a stronger case can be put: the Aristotelian Stadium, as it stands, is not immensely impressive; but it points to a crucial feature in the concept of locomotion. In short, the Aristotelian Stadium is philosophically important, and not ‘very silly’.

I offer the following argument as a refurbished account of the Stadium. ‘Our concept of motion is intimately connected with the two continua of space and time. The concept is linked logically to those two notions in a variety of ways; two of the most important are mirrored in these two implications:

- (1) If *a* moves past a sequence of *n* *Fs* and each *F* is *k* units long, then *a* moves *nk* units.
- (2) If *a* moves for a period of *T* units at a constant speed of *j.u.p.u.*, and covers *m* units in that time, then $T=m/j$ units.

Now consider *fig. 5* and *fig. 2.1*. Substitute *B*₁ for *a* in (1) and (2): it is easy to show, by two applications of (1) and (2), that *T*, the period from *t*₁ to *t*₂, is equal to $\frac{1}{2}T$. For the passage of *B*₁ past the single *A* gives $T=1k/j$. And the passage of *B*₁ past the two *Cs* gives $T=2k/j$. Thus the concept of motion implies (1) and (2); and (1) and (2) lead to contradiction. Hence the concept of motion is incoherent. Hence nothing can move.’

Such a reconstruction has two points in its favour. First, it relates the Stadium to Zeno’s general attack on motion; most accounts do not explain the connexion between the fourth paradox of motion and Zeno’s overall aim. Second, it provides an interesting argument against motion. For I am inclined to think that both (1) and (2) would have commended themselves to Zeno’s contemporaries, and do commend themselves to us in our unthinking moments. Our conception of locomotion is such that (1) and (2) seem to be implicit in it; yet (1) and (2) lead to unacceptable results. Zeno infers, I suggest, that the concept of motion is inconsistent; we may prefer to say that men who subscribe to (1) and (2) confusedly grasp the consistent concept of motion. The mode of expression is immaterial: if we are to move in the Stadium we must articulate a concept of motion which does not give rise to (1) and (2).

Aristotle's answer to Zeno may now seem as unsatisfactory as in reality it is: according to Aristotle, Zeno ignores the fact that the *As* are stationary and the *Bs* moving. The criticism is not profound: where, we may ask, does Aristotle's objection fit into the argument based on (1) and (2)? How would he reject those twin principles? What notion of movement would he supply which avoids (1) and (2)? I do not say that those questions are unanswerable; I do say that they are unanswered, and that that fact makes Aristotle's reply to Zeno inadequate.

An adequate reply to Zeno, and an adequate account of motion, would focus on the truth that motion is a relational thing: '...it doth not appear to me that there can be any motion other than *relative*: so that to conceive motion, there must be conceived two bodies, whereof the distance or position in regard to each other is varied. Hence if there was only one body in being, it could not possibly be moved. This seems evident, in that the idea I have of motion doth necessarily include relation' (Berkeley, *Principles of Human Knowledge*, § 112).

Motion, in brief, is relative not absolute. That slogan may mislead; and it is worth stating clearly two things that it does not imply. First, it does not imply the incoherence of 'absolute' motion in Newton's sense: Newtonian 'absolute' motion is merely a privileged form of relative motion; for according to Newton a body is in 'absolute' motion if it is changing position relative to absolute space. (It is a further question whether anything can be made of absolute space.) Second, the rejection of 'absolute' motion does not require a wholesale change in our everyday language of movement. For though we do regularly speak as though motion were absolute, it is fairly simple to translate such talk into talk of motion relative to our human inertial frame, the earth: 'he's moving' can regularly be taken as elliptical for 'he's changing position relative to (some point on) the earth's surface'. (Regularly, but not, of course, always: in the enclosed world of an aeroplane or a ship we naturally take the enclosing vehicle as our reference frame; and Christmas cracker puzzles, such as the conundrum of the man running round the monkey, remind us that all is not plain sailing here.)

To say that motion is relative is to say no more than this: '*a* moves' only makes sense if it is taken as elliptical for some two-place predication of the form '*a* changes position relative to *b*'. '*a* moves' is comparable to '*a* leads' or '*a* follows'; leading and following are relational notions, even though we often and intelligibly use the verbs without explicitly mentioning a *relatum*: and just as '*a* leads' is elliptical for '*a* leads *b*' so is '*a* moves' elliptical for '*a* moves *vis-à-vis* *b*'. There is, of course, nothing unusual in having such elliptical, absolute-seeming, uses for essentially relational verbs; in the case of 'move', however, there is a peculiar danger of taking the absolute-seeming use at face value, and inventing an absolute quality of motion to serve as *denotatum* for the absolute verb. (I suspect that this connects with the old Aristotelian view that motion is immediately perceptible, a 'simple idea': if we can simply *see* that a thing is in motion, surely motion is an intrinsic quality and not a relation?)

What bearing has the relativity of motion on the Stadium? It allows us to retain the essence of implications (1) and (2) while rejecting their cruel accidents. Thus, to take the case of (1), we shall have:

(1*) If *a* moves past a sequence of *n* *Fs* and each *F* is *k* units long, then relative to those *Fs* *a* moves *nk* units.

The plausibility of (1) carries over to (1*) and (1*) contains the truth which (1) strove in vain to express. Moreover (1*) has no hard consequences, once we realize that there is no contradiction between ‘*a* moves *n* units relative to *b*’ and ‘*a* moves *m* units relative to *c*’ (where $b \neq c$ and $n \neq m$). For I may move relative to one thing while resting relative to another—there are a thousand everyday examples of that.

I do not claim that only a relative concept of motion will answer Zeno’s argument; but that such a concept is both correct in itself and sufficient to answer Zeno seems clear enough. If Zeno’s Stadium encourages us to clarify our conception of motion in that way, then that, I submit, is enough to make an honest argument of it. Zeno, of course, did not intend his Stadium to refute ‘absolute’ motion and elicit a relativized concept; and indeed he does not seem to have stirred any of his successors to develop such a sanitary notion. For all that, the Stadium does show, in a pointed and pregnant way, the need for a subtler concept of locomotion than we are apt at first sight to formulate. And that turns the paradox of the Stadium, in its plain Aristotelian form, into an argument of some significance.

(i) *Two last remarks*

I end my long consideration of Zeno’s paradoxes with two general comments.

First, what, we may wonder, is the connexion between the four paradoxes on motion and the Platonic schema into which I attempted to fit the paradoxes of plurality? We may always suppose that the arguments about motion, which Aristotle treats as a unitary group and which according to Elias constituted an independent Zenonian volume, were in their original form a separate publication. We might indeed conjecture that they were worked out later in Zeno’s life, the fruits of his intellectual maturity.

On the other hand, Plato names ‘at rest and moving’ as one of the set of opposites which Zeno proved to hold of ‘the many’ (*Phaedrus* 261D=A 13). Thus one case of the Platonic scheme will be: (Z7) (a) If *P*, then everything moves, and (b) if *P*, then nothing moves. Here (Z7a) will have required no special argument: evidently, the ‘many things’ which the pluralists admit do, all of them, move. And it is not vastly implausible to see the four paradoxes of motion as intended to support (Z7b). In that case, the paradoxes will, originally, have appeared in Zeno’s juvenile book, alongside the other arguments I have dissected; and some later editor will have assembled them in a little treatise of their own, preserving their philosophical content while divesting them of their original polemical form.

Second, what general portrait of Zeno emerges from these two chapters? My discussion has, I think, confirmed the view I sketched on an earlier page (above, p. 236). Zeno ‘put forward no view of his own but puzzled further about’ Eleatic issues (pseudo-Plutarch, A 23). Zeno was no original philosopher; he is not a member of the long line of thinkers stretching from Thales to Melissus, men of vast learning, wide pretensions, profound insights. Rather, he puzzled: negative, destructive, polemical, Zeno was the first of the ‘Sophists’. His aims were critical, not constructive; his methods subtle not solid. Yet from his Sophist’s quiver he drew a few darts of brilliance and acuity; and those darts have made him a prince of philosophers *malgré lui*.